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SUMMER– 18 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code: 22210**

Important Instructions to examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.		**Solve any FIVE of the following:**	**10**
	a)	If $f(x)=64^{x}+\log_3 x$, find $f\left(\frac{1}{3}\right)$	**02**
	Ans	$f\left(\frac{1}{3}\right)=(64)^{\frac{1}{3}}+\log_3\left(\frac{1}{3}\right)$	½
		$\therefore f\left(\frac{1}{3}\right)=4-\log_3 3$	½
		$\therefore f\left(\frac{1}{3}\right)=4-1$	½
		$\therefore f\left(\frac{1}{3}\right)=3$	½
		--	
	b)	If $f(x)=\sin x$, show that $f(3x)=3f(x)-4f^{3}(x)$	**02**
	Ans	$3f(x)-4f^{3}(x)$	
		$=3\sin x-4\sin^{3}x$	½
		$=\sin 3x$	1
		$=f(3x)$	½
		OR	
		$f(3x)$	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	b)	$= \sin 3x$	½
		$= 3\sin x - 4\sin^3 x$	1
		$= 3f(x) - 4f^3(x)$	½
	c)	Find $\frac{dy}{dx}$ if $y = e^x \sin^{-1} x$	**02**
	Ans	$y = e^x \sin^{-1} x$	
		$\therefore \frac{dy}{dx} = e^x \frac{1}{\sqrt{1-x^2}} + \sin^{-1} x\ e^x$	1+1
		$\therefore \frac{dy}{dx} = e^x \left(\frac{1}{\sqrt{1-x^2}} + \sin^{-1} x \right)$	
	d)	Evaluate: $\int x(x-1)^2 dx$	**02**
	Ans	$\int x(x-1)^2 dx$	
		$= \int x(x^2 - 2x + 1)\ dx$	½
		$= \int (x^3 - 2x^2 + x)\ dx$	½
		$= \frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2} + c$	1
	e)	Evaluate: $\int \sin^2 2x\ dx$	**02**
	Ans	$\int \sin^2 2x\ dx$	
		$= \frac{1}{2}\int 2\sin^2 2x\ dx$	½
		$= \frac{1}{2} \int (1 - \cos 4x)\ dx$	½
		$= \frac{1}{2}\left(x - \frac{\sin 4x}{4} \right) + c$	1
	f)	Find the area bounded by the curve $y = x^2$, x-axis and ordinates $x = 0$ to $x = 3$	**02**

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Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	f) Ans	Area $A=\int_a^b y\ dx$	
		$=\int_0^3 x^2 dx$	½
		$=\left[\frac{x^3}{3}\right]_0^3$	½
		$=\left(\frac{3^3}{3}-0\right)$	½
		$=9$	½
	g)	Express $z=\frac{1-i}{1+i}$ in $a+ib$ form , where $i=\sqrt{-1}$ and a,b are real number	**02**
	Ans	$z=\frac{1-i}{1+i}$	
		$\therefore z=\frac{1-i}{1+i}\times\frac{1-i}{1-i}$	½
		$\therefore z=\frac{1-2i+i^2}{1^2-i^2}$	½
		$\therefore z=\frac{1-2i-1}{1+1}$	½
		$\therefore z=\frac{-2i}{2}$	
		$\therefore z=-i=0-i$	½
2.		**Attempt any THREE of the following:**	**12**
	a)	If $13x^2+2x^2y+y^3=1$, find $\frac{dy}{dx}$ at $(1,-2)$	**04**
	Ans	$13x^2+2x^2y+y^3=1$	
		$\therefore 26x+2\left(x^2\frac{dy}{dx}+y2x\right)+3y^2\frac{dy}{dx}=0$	1
		$\therefore 26x+2x^2\frac{dy}{dx}+4xy+3y^2\frac{dy}{dx}=0$	

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Q. No.	Sub Q. N.	Answers	Marking Scheme
2.	a)	$\therefore 2x^2\frac{dy}{dx}+3y^2\frac{dy}{dx}=-26x-4xy$	1
		$\therefore \left(2x^2+3y^2\right)\frac{dy}{dx}=-26x-4xy$	
		$\therefore \frac{dy}{dx}=\frac{-26x-4xy}{2x^2+3y^2}$	1
		at $\left(1,-2\right)$	
		$\frac{dy}{dx}=\frac{-18}{14}=\frac{-9}{7}$	1
	b)	If $x=a\left(\theta+\sin\theta\right)$, $y=a\left(1-\cos\theta\right)$, find $\frac{dy}{dx}$ at $\theta=\frac{\pi}{2}$	**04**
	Ans	$x=a\left(\theta+\sin\theta\right)$ $\quad y=a\left(1-\cos\theta\right)$	
		$\frac{dx}{d\theta}=a\left(1+\cos\theta\right)$ $\quad \frac{dy}{d\theta}=a\left(0+\sin\theta\right)=a\sin\theta$	1+1
		$\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{a\sin\theta}{a\left(1+\cos\theta\right)}$	1
		$\frac{dy}{dx}=\frac{\sin\theta}{1+\cos\theta}$	
		at $\theta=\frac{\pi}{2}$	
		$\therefore \frac{dy}{dx}=\frac{\sin\frac{\pi}{2}}{1+\cos\frac{\pi}{2}}=1$	1
	c)	The rate of working of an engine is given by the expression $10V+\frac{4000}{V}$, where V is the speed of the engine.Find the speed at which the rate of working is the least.	**04**
	Ans	The rate of working is, $W=10V+\frac{4000}{V}$	
		$\therefore \frac{dW}{dV}=10-\frac{4000}{V^2}$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	c)	$\therefore \frac{d^2W}{dV^2} = \frac{8000}{V^3}$	½
		Consider $\frac{dW}{dV} = 0$	½
		$\therefore 10 - \frac{4000}{V^2} = 0$	
		$\therefore 10 = \frac{4000}{V^2}$	½
		$\therefore V^2 = 400$	
		$\therefore V = 20, -20$	1
		at $V = 20$	
		$\therefore \frac{d^2W}{dV^2} = \frac{8000}{(20)^3} = 1 > 0$	1
		$\therefore$ The speed is $V = 20$ at which the rate of working is least	
		--	
	d)	A telegraph wire hangs in the form of a curve $y = 2\sin x - \sin 2x$. Find the radius of curvature of the wire at the point $x = \frac{\pi}{2}$	**04**
	Ans	$y = 2\sin x - \sin 2x$	
		$\therefore \frac{dy}{dx} = 2\cos x - 2\cos 2x$	½
		$\therefore \frac{d^2y}{dx^2} = -2\sin x + 4\sin 2x$	½
		at $x = \frac{\pi}{2}$	
		$\therefore \frac{dy}{dx} = 2\cos\left(\frac{\pi}{2}\right) - 2\cos 2\left(\frac{\pi}{2}\right) = 2$	½
		$\therefore \frac{d^2y}{dx^2} = -2\sin\left(\frac{\pi}{2}\right) + 4\sin 2\left(\frac{\pi}{2}\right) = -2$	½
		$\therefore$ Radius of curvature $= \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[1+(2)^2\right]^{3/2}}{-2}$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	d)	$\therefore$ Radius of curvature $=-5.59$ i.e. 5.59	1
		--	
3.		**Solve any <u>THREE</u> of the following:**	**12**
	a)	Find the equation of the tangent to the curve $y=9x^2-12x+7$ which is parellel to the $x-$axis	**04**
	Ans	$y=9x^2-12x+7$	
		$\therefore \frac{dy}{dx}=18x-12$	½
		tangent is parellel to the $x-$axis	
		$\therefore \frac{dy}{dx}=0$	
		$\therefore 18x-12=0$	½
		$\therefore x=\frac{2}{3}$	½
		$\therefore y=3$	½
		$\therefore (x_1,y_1)=\left(\frac{2}{3},3\right)$	
		$\therefore$ slope of tangent , $m=0$	½
		Equation of tangent at $\left(\frac{2}{3},3\right)$ is	
		$y-3=0\left(x-\frac{2}{3}\right)$	1
		$\therefore y-3=0$	½
		--	
	b)	Find $\frac{dy}{dx}$ if $y=\log\left(\frac{\sin x}{1+\cos x}\right)$	**04**
	Ans	$y=\log\left(\frac{\sin x}{1+\cos x}\right)$	
		$\therefore \frac{dy}{dx}=\frac{1}{\frac{\sin x}{1+\cos x}}\frac{d}{dx}\left(\frac{\sin x}{1+\cos x}\right)$	1
		$\therefore \frac{dy}{dx}=\frac{1+\cos x}{\sin x}\left(\frac{(1+\cos x)\cos x-\sin x(0-\sin x)}{(1+\cos x)^2}\right)$	1

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Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	b)	$\therefore \frac{dy}{dx}=\frac{1}{\sin x}\left(\frac{\cos x+\cos^2 x+\sin^2 x}{1+\cos x}\right)$	1
		$\therefore \frac{dy}{dx}=\frac{1}{\sin x}\left(\frac{\cos x+1}{1+\cos x}\right)$	1
		$\therefore \frac{dy}{dx}=\frac{1}{\sin x}=\cos ecx$	
		OR	
		$y=\log\left(\frac{\sin x}{1+\cos x}\right)$	
		$\therefore y=\log\left(\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$	1
		$\therefore y=\log\left(\tan\frac{x}{2}\right)$	1
		$\therefore \frac{dy}{dx}=\frac{1}{\tan\frac{x}{2}}\left(\sec^2\frac{x}{2}\right)\left(\frac{1}{2}\right)$	2
		$\therefore \frac{dy}{dx}=\frac{\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos^2\frac{x}{2}}=\frac{1}{2\sin\frac{x}{2}\cos\frac{x}{2}}$	
		$\therefore \frac{dy}{dx}=\frac{1}{\sin x}=\cos ecx$	
		OR	
		$y=\log\left(\frac{\sin x}{1+\cos x}\right)$	
		$\therefore y=\log(\sin x)-\log(1+\cos x)$	1
		$\therefore \frac{dy}{dx}=\frac{1}{\sin x}\cos x-\frac{1}{1+\cos x}(-\sin x)$	1
		$\therefore \frac{dy}{dx}=\frac{\cos x(1+\cos x)}{\sin x}+\frac{\sin x}{1+\cos x}$	½
		$\therefore \frac{dy}{dx}=\frac{\cos x+\cos^2 x+\sin^2 x}{\sin x(1+\cos x)}$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	b)	$\therefore \frac{dy}{dx} = \frac{\cos x + 1}{\sin x(1+\cos x)} = \frac{1}{\sin x} = \operatorname{cos}ecx$	½
		--	
	c)	If $x^y = e^{x-y}$, then prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$	**04**
	Ans	$x^y = e^{x-y}$	
		$\log x^y = \log e^{x-y}$	½
		$y\log x = (x-y)\log e$	½
		$\therefore y\log x = x - y$	
		$\therefore y\log x + y = x$	½
		$\therefore y(\log x + 1) = x$	
		$\therefore y = \frac{x}{\log x + 1}$	½
		$\therefore \frac{dy}{dx} = \frac{(\log x+1)\frac{d}{dx}(x) - x\frac{d}{dx}(\log x+1)}{(\log x+1)^2}$	1
		$\therefore \frac{dy}{dx} = \frac{(\log x+1)\cdot 1 - x\left(\frac{1}{x}+0\right)}{(\log x+1)^2}$	
		$\therefore \frac{dy}{dx} = \frac{\log x + 1 - 1}{(\log x+1)^2}$	
		$\therefore \frac{dy}{dx} = \frac{\log x}{(\log x+1)^2}$	1
		--	
	d)	Evaluate : $\int \frac{\cos x}{1+\sin^2 x}dx$	**04**
	Ans	Put $\sin x = t$	1
		$\therefore \cos x dx = dt$	1
		$= \int \frac{dt}{1+t^2}$	
		$= \tan^{-1} t + c$	1
		$= \tan^{-1}(\sin x) + c$	1

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Q. No.	Sub Q.N.	Answers	Marking Scheme
4.		**Solve any THREE of the following:**	**12**
	a)	Evaluate: $\int \frac{\log x}{x(2+\log x)} \frac{dx}{(3+\log x)}$	**04**
	Ans	$\int \frac{\log x}{x(2+\log x)(3+\log x)} dx$	
		Put $\log x = t$	
		$\therefore \frac{1}{x} dx = dt$	½
		$\int \frac{t}{(2+t)(3+t)} dt$	
		consider $\frac{t}{(2+t)(3+t)} = \frac{A}{2+t} + \frac{B}{3+t}$	½
		$\therefore t = A(3+t) + B(2+t)$	
		Put $t=-2$	
		$A=-2$	½
		Put $t=-3$	
		$B=3$	½
		$\therefore \frac{t}{(2+t)(3+t)} = \frac{-2}{2+t} + \frac{3}{3+t}$	½
		$\therefore \int \frac{t}{(2+t)(3+t)} dt = \int \left(\frac{-2}{2+t} + \frac{3}{3+t} \right) dt$	
		$= -2\log(2+t) + 3\log(3+t) + c$	1
		$= -2\log(2+\log x) + 3\log(3+\log x) + c$	½
		--	
4.	b)	Evaluate: $\int \frac{dx}{3-2\sin x}$	**04**
	Ans	$\int \frac{dx}{3-2\sin x}$	
		Put $\tan\frac{x}{2} = t$, $dx = \frac{2dt}{1+t^2}$, $\sin x = \frac{2t}{1+t^2}$	
		$\int \frac{\frac{2dt}{1+t^2}}{3-2\left(\frac{2t}{1+t^2}\right)}$	1

Q. No.	Sub Q. N.	Answers	Marking Scheme
4.	b)	$=2\int\frac{dt}{3\left(1+t^2\right)-2\left(2t\right)}$	
		$=2\int\frac{dt}{3+3t^2-4t}$	
		$=2\int\frac{dt}{3t^2-4t+3}$	½
		$=\frac{2}{3}\int\frac{dt}{t^2-\frac{4}{3}t+1}$	½
		$T.T.=\left(\frac{1}{2}\times\frac{4}{3}\right)^2=\frac{4}{9}$	
		$=\frac{2}{3}\int\frac{dt}{t^2-\frac{4}{3}t+\frac{4}{9}-\frac{4}{9}+1}$	½
		$=\frac{2}{3}\int\frac{dt}{\left(t-\frac{2}{3}\right)^2+\frac{5}{9}}$	
		$=\frac{2}{3}\int\frac{dt}{\left(t-\frac{2}{3}\right)^2+\left(\frac{\sqrt{5}}{3}\right)^2}$	½
		$=\frac{2}{3}\frac{1}{\frac{\sqrt{5}}{3}}\tan^{-1}\left(\frac{t-\frac{2}{3}}{\frac{\sqrt{5}}{3}}\right)+c$	½
		$=\frac{2}{\sqrt{5}}\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)-\frac{2}{3}}{\frac{\sqrt{5}}{3}}\right)+c$	½
		--	
	c)	Evaluate: $\int\frac{x\sin^{-1}x}{\sqrt{1-x^2}}\,dx$	**04**
	Ans	$\int\frac{x\sin^{-1}x}{\sqrt{1-x^2}}\,dx$	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	c)	Put $\sin^{-1} x = t \quad \therefore x = \sin t$	
		$\therefore \frac{1}{\sqrt{1-x^2}} dx = dt$	1
		$\therefore \int \sin t \; t \; dt$	
		$= \int t \sin t \; dt$	
		$= t(-\cos t) - \int(-\cos t) 1 dt$	1
		$= -t\cos t + \int \cos t \; dt$	
		$= -t\cos t + \sin t + c$	1
		$= -\sin^{-1} x \cos\left(\sin^{-1} x\right) + x + c$ OR $-\sin^{-1} x\sqrt{1-x^2} + x + c$	1
		--	
	d)	Evaluate: $\int \frac{x+1}{x^2(x-2)} dx$	**04**
	Ans	$\int \frac{x+1}{x^2(x-2)} dx$	
		Consider $\frac{x+1}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$	½
		$\therefore x+1 = Ax(x-2) + B(x-2) + Cx^2$	
		Put $x = 0$	
		$\therefore B = -\frac{1}{2}$	½
		Put $x = 2$	
		$\therefore C = \frac{3}{4}$	½
		Put $x = 1$	
		$2 = -A - B + C$	
		$\therefore 2 = -A + \frac{1}{2} + \frac{3}{4}$	
		$\therefore A = \frac{-3}{4}$	½
		$\frac{x+1}{x^2(x-2)} = \frac{\frac{-3}{4}}{x} + \frac{\frac{-1}{2}}{x^2} + \frac{\frac{3}{4}}{x-2}$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	d)	$\therefore \int \frac{x+1}{x^2(x-2)}dx = \int \left(\frac{\frac{-3}{4}}{x} + \frac{\frac{-1}{2}}{x^2} + \frac{\frac{3}{4}}{x-2} \right) dx$	½
		$\therefore \int \frac{x+1}{x^2(x-2)}dx = \frac{-3}{4}\log x + \frac{1}{2x} + \frac{3}{4}\log(x-2) + c$	1
	e)	Evaluate: $\int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\, dx$	**04**
	Ans	$I = \int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\, dx$ --------------------(1)	
		$I = \int_1^3 \frac{\sqrt[3]{(1+3-x)+5}}{\sqrt[3]{(1+3-x)+5}+\sqrt[3]{9-(1+3-x)}}\, dx$	1
		$\therefore I = \int_1^3 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+5}}\, dx$ --------------------(2)	
		add (1) and (2)	
		$I + I = \int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\, dx + \int_1^3 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+5}}\, dx$	
		$2I = \int_1^3 \frac{\sqrt[3]{x+5}+\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+5}}\, dx$	1
		$2I = \int_1^3 1\, dx$	½
		$2I = [x]_1^3$	1
		$2I = 3-1$	
		$I = 1$	½
5.		**Solve any <u>TWO</u> of the following:**	**12**
	a)	Find the area enclosed between the parabola $y = x^2$ and the line $y = 4$.	**04**
	Ans	$y = x^2$	
		$4 = x^2$	
		$\therefore x = \pm 2$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	a)	$\therefore A=\int_{-2}^{2}\left(x^2-4\right)$	½
		$A=\left(\frac{x^3}{3}-4x\right)_{-2}^{2}$	1
		$A=\left(\frac{(2)^3}{3}-4(2)\right)-\left(\frac{(-2)^3}{3}-4(-2)\right)$	1
		$\therefore A=\frac{16}{3}-16$	
		$\therefore A=\frac{32}{3}$ or 10.667	1
		--	
	b)	**Attempt the following:**	**06**
	(i)	Find the order and degree of the differential equation $\frac{d^2y}{dx^2}=\left(y+\frac{dy}{dx}\right)^{3/2}$	**02**
	Ans	$\frac{d^2y}{dx^2}=\left(y+\frac{dy}{dx}\right)^{3/2}$	
		squaring	
		$\left(\frac{d^2y}{dx^2}\right)^2=\left(y+\frac{dy}{dx}\right)^3$	
		Order of D.E. $=2$	1
		Degree of D.E. $=2$	1
		--	
	ii)	Solve: $x\frac{dy}{dx}-y=x^2$	**04**
	Ans	$x\frac{dy}{dx}-y=x^2$	
		Divide by x	
		$\frac{dy}{dx}-\frac{y}{x}=x$	½
		$\therefore$ Comparing with $\frac{dy}{dx}+Py=Q$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	b)(ii)	$P=\frac{-1}{x}$, $Q=x$	½
		Integrating factor $IF=e^{\int\frac{-1}{x}dx}=e^{-\log x}=\frac{1}{x}$	1
		$y.IF=\int Q.IFdx+c$	
		$y\frac{1}{x}=\int x.\frac{1}{x}dx$	
		$\frac{y}{x}=\int 1dx$	1
		$\frac{y}{x}=x+c$	1
	c)	The current 'i' is given by $L\frac{di}{dt}=30\sin(10\pi t)$, where L is inductance and t is time. Find 'i' in terms of t, given that $L=2$ and $i=0$ at $t=0$	**04**
	Ans	$Ldi=30\sin(10\pi t)dt$	½
		$\int Ldi=\int 30\sin(10\pi t)dt$	
		$Li=30\left(\frac{-\cos(10\pi t)}{10\pi}\right)+c$	2
		$Li=\frac{-3\cos(10\pi t)}{\pi}+c$	½
		at $t=0, i=0$	
		$L(0)=\frac{-3\cos(0)}{\pi}+c$	
		$0=\frac{-3}{\pi}+c$	
		$\therefore c=\frac{3}{\pi}$	1
		$\therefore Li=\frac{-3\cos(10\pi t)}{\pi}+\frac{3}{\pi}$	½
		at $L=2$	
		$\therefore 2i=\frac{-3\cos(10\pi t)}{\pi}+\frac{3}{\pi}$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	c)	$\therefore i=\frac{3}{2\pi}\left(-\cos\left(10\pi t\right)+1\right)$	1
		--	
6.		**Solve any <u>TWO</u> of the following:**	**12**
	a)	**Attempt the following:**	**06**
	(i)	If $z_1=-3+4i$, $z_2=5-3i$ express $\frac{z_1}{z_2}$ in $x+iy$ form.	**03**
	Ans	$\frac{z_1}{z_2}=\frac{-3+4i}{5-3i}$	
		$\therefore\frac{z_1}{z_2}=\frac{-3+4i}{5-3i}\times\frac{5+3i}{5+3i}$	½
		$\therefore\frac{z_1}{z_2}=\frac{-15-9i+20i+12i^2}{25-9i^2}$	1
		$\therefore\frac{z_1}{z_2}=\frac{-15-9i+20i+12\left(-1\right)}{25-9\left(-1\right)}$	½
		$\therefore\frac{z_1}{z_2}=\frac{-27+11i}{34}$	½
		$\therefore\frac{z_1}{z_2}=\frac{-27}{34}+\frac{11}{34}i$	½
		--	
	(ii)	Find $\mathrm{L}\left\{e^{-3t}\sin 2t\right\}$	**03**
	Ans	$\mathrm{L}\left\{e^{-3t}\sin 2t\right\}$	
		$L\left\{\sin 2t\right\}=\frac{2}{s^2+2^2}=\frac{2}{s^2+4}$	½
		$\therefore\mathrm{L}\left\{e^{-3t}\sin 2t\right\}=\frac{2}{\left(s+3\right)^2+4}$	1
		$\therefore\mathrm{L}\left\{e^{-3t}\sin 2t\right\}=\frac{2}{s^2+6s+9+4}$	1
		$\therefore\mathrm{L}\left\{e^{-3t}\sin 2t\right\}=\frac{2}{s^2+6s+13}$	½
		--	

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	b)	Find $L^{-1}\left\{\frac{3s+1}{(s-1)(s^2+1)}\right\}$	**06**
	Ans	Let	
		$\frac{3s+1}{(s-1)(s^2+1)}=\frac{A}{s-1}+\frac{Bs+C}{s^2+1}$	½
		$3s+1=(s^2+1)A+(s-1)(Bs+C)$	
		Put $s=1$	
		$\therefore A=2$	½
		Put $s=0$	
		$1=A+(-1)C$	
		$\therefore 1=2-C$	
		$\therefore C=1$	½
		Put $s=-1$	
		$-2=2A+(-2)(-B+C)$	
		$\therefore -2=2(2)+2B-2(1)$	
		$\therefore -2=2+2B$	
		$\therefore B=-2$	½
		$\therefore \frac{3s+1}{(s-1)(s^2+1)}=\frac{2}{s-1}+\frac{-2s+1}{s^2+1}$	½
		$\therefore L^{-1}\left\{\frac{3s+1}{(s-1)(s^2+1)}\right\}=2L^{-1}\left\{\frac{1}{s-1}\right\}-2L^{-1}\left\{\frac{s}{s^2+1}\right\}+L^{-1}\left\{\frac{1}{s^2+1}\right\}$	½
		$=2e^t-2\cos t+\sin t$	1+1+1
		--	
	c)	Solve the differential equation using Laplace transform.	**06**
		$L\frac{di}{dt}+Ri=V$, $i(0)=0$	
	Ans	$L\frac{di}{dt}+Ri=V$	
		Apply laplace transform on both sides,	

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	c)	$\therefore L\left\{L\frac{di}{dt}+Ri\right\}=L\{V\}$	
		$\therefore L\left\{L\left(\frac{di}{dt}\right)\right\}+R\{L(i)\}=VL\{1\}$	½
		$\therefore L\{sL(i)-i(0)\}+R\{L(i)\}=V\left(\frac{1}{s}\right)$	1
		$\therefore L\{sL(i)-0\}+R\{L(i)\}=V\left(\frac{1}{s}\right)$	
		$\therefore (Ls+R)L(i)=\frac{V}{s}$	½
		$\therefore L(i)=\frac{V}{s(Ls+R)}$	
		$\therefore L(i)=\frac{V}{L}\frac{1}{s\left(s+\frac{R}{L}\right)}$	½
		Partial fraction is	
		$\frac{1}{s\left(s+\frac{R}{L}\right)}=\frac{A}{s}+\frac{B}{s+\frac{R}{L}}$	½
		$1=\left(s+\frac{R}{L}\right)A+sB$	
		Put $s=0$	
		$\therefore A=\frac{L}{R}$	½
		Put $s=-\frac{R}{L}$	
		$\therefore B=-\frac{L}{R}$	½
		$\therefore \frac{1}{s\left(s+\frac{R}{L}\right)}=\frac{\frac{L}{R}}{s}+\frac{-\frac{L}{R}}{s+\frac{R}{L}}=\frac{L}{R}\left(\frac{1}{s}-\frac{1}{s+\frac{R}{L}}\right)$	½
		$\therefore L(i)=\frac{V}{L}\frac{L}{R}\left(\frac{1}{s}-\frac{1}{s+\frac{R}{L}}\right)$	½

Q. No.	Sub Q. N.	Answers	Marking Scheme
6.	c)	$\therefore i=\frac{V}{R}L^{-1}\left\{\frac{1}{s}-\frac{1}{s+\frac{R}{L}}\right\}$	½
		$\therefore i=\frac{V}{R}\left(1-e^{-\frac{R}{L}t}\right)$	½

		Important Note	
		In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.	

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WINTER– 18 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code: 22210**

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.		**Solve any FIVE of the following:**	**10**
	a)	If $f(x)=3x^2-5x+7$ show that $f(-1)=3f(1)$	**02**
	Ans	$f(-1)=15$	1
		$f(1)=5$	½
		$\therefore 3f(1)=15$	½
		$\therefore f(-1)=3f(1)$	
		--	
	b)	Define odd and even function with suitable examples.	**02**
	Ans	If $f(-x)=-f(x)$ then the function is an odd function	½
		$e.g. f(x)=x^3+x$	½
		$f(-x)=(-x)^3+(-x)$	
		$\therefore f(-x)=-(x^3+x)$	
		$\therefore f(-x)=-f(x)$	
		If $f(-x)=f(x)$ then the function is an even function	½
		$e.g. f(x)=x^2+1$	½
		$\therefore f(-x)=(-x)^2+1$	
		$\therefore f(-x)=x^2+1$	
		$\therefore f(-x)=f(x)$	
		--	

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	c)	Find dy/dx if $y = a^x + x^a + a^a + \sqrt{x}$	**02**
	Ans	$y = a^x + x^a + a^a + \sqrt{x}$	
		$\therefore \frac{dy}{dx} = a^x \log a + ax^{a-1} + 0 + \frac{1}{2\sqrt{x}}$	2
		$\therefore \frac{dy}{dx} = a^x \log a + ax^{a-1} + \frac{1}{2\sqrt{x}}$	
	d)	Evaluate $\int 1/x^2+4 \; dx$	**02**
	Ans	$\int \frac{1}{x^2+4} \; dx$	
		$= \int \frac{1}{x^2+(2)^2} \; dx$	½
		$= \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$	1½
	e)	Evaluate $\int x.e^x dx$	**02**
	Ans	$\int x.e^x dx$	
		$= x\left(\int e^x dx\right) - \int\left(\int e^x dx \frac{d}{dx}(x)\right)dx$	½
		$= xe^x - \int e^x.1 \; dx$	½
		$= xe^x - \int e^x \; dx$	½
		$= xe^x - e^x + c$	½
	f)	If $z_1 = 4-5i$ and $z_2 = 3+7i$ find $\lvert z_1 + z_2 \rvert$	**02**
	Ans	$z_1 + z_2 = 4 - 5i + 3 + 7i$	
		$\therefore z_1 + z_2 = 7 + 2i$	1
		$\therefore \lvert z_1 + z_2 \rvert = \sqrt{(7)^2 + (2)^2}$	
		$\therefore \lvert z_1 + z_2 \rvert = \sqrt{49+4}$	
		$\therefore \lvert z_1 + z_2 \rvert = \sqrt{53}$	1

WINTER– 18 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code: 22210**

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	g)	Find the area bounded by the curve $y=3x^2$, the lines $x=1$, $x=3$ and $x-$axis	**02**
	Ans	Area $A=\int_a^b y\ dx$	
		$=\int_1^3 3x^2 dx$	
		$=3\left[\frac{x^3}{3}\right]_1^3$	1
		$=3\left(\frac{3^3}{3}-\frac{1^3}{3}\right)$	
		$=26$	1
2.		**Solve any THREE of the following:**	**12**
	a)	Find dy/dx if $x^3+y^3=3axy$	**04**
	Ans	$x^3+y^3=3axy$	
		$\therefore 3x^2+3y^2\frac{dy}{dx}=3a\left(x\frac{dy}{dx}+y.1\right)$	2
		$\therefore 3x^2+3y^2\frac{dy}{dx}=3ax\frac{dy}{dx}+3ay$	½
		$\therefore \left(3y^2-3ax\right)\frac{dy}{dx}=3ay-3x^2$	½
		$\therefore \frac{dy}{dx}=\frac{3ay-3x^2}{3y^2-3ax}$	
		$\therefore \frac{dy}{dx}=\frac{ay-x^2}{y^2-ax}$	1
	b)	Find dy/dx if $x=1/t$ and $y=1-1/t$	**04**
	Ans	$x=1/t$ and $y=1-1/t$	
		$\therefore \frac{dx}{dt}=-1/t^2$ and $\frac{dy}{dt}=1/t^2$	1+1
		$\therefore \frac{dy}{dx}=\frac{dy/dt}{dx/dt}$	

Q. No.	Sub Q. N.	Answers	Marking Scheme
2.	b)	$\therefore \frac{dy}{dx} = \frac{1/t^2}{-1/t^2}$	1
		$\therefore \frac{dy}{dx} = -1$	1
	c)	A bullet is fired into a mud bank and penetrates $(120t - 3600t^2)$ m. in $'t'$ sec. after impact. Calculate maximum depth of penetration.	**04**
	Ans	Let $s = 120t - 3600t^2$	
		$\therefore \frac{ds}{dt} = 120 - 7200t$	1
		$\therefore \frac{d^2s}{dt^2} = -7200$	1
		Consider $\frac{ds}{dt} = 0$	
		$\therefore 120 - 7200t = 0$	
		$\therefore 120 = 7200t$	
		$\therefore t = \frac{1}{60}$	1
		at $t = \frac{1}{60}$	
		$\therefore \frac{d^2s}{dt^2} = -7200 < 0$	½
		$\therefore$ The maximum depth of penetration is,	
		$s = 120\left(\frac{1}{60}\right) - 3600\left(\frac{1}{60}\right)^2$	
		$\therefore s = 1$ meter	½
	d)	Find radius of curvature to the curve $y = x^3$ at $(2,8)$	**04**
	Ans	$y = x^3$	
		$\therefore \frac{dy}{dx} = 3x^2$	1
		$\therefore \frac{d^2y}{dx^2} = 6x$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	d)	at $(2,8)$	
		$\frac{dy}{dx}=3(2)^2=12$	½
		$\frac{d^2y}{dx^2}=6(2)=12$	½
		$\therefore$ Radius of curvature is $\rho=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$	
		$\therefore \rho=\frac{\left[1+(12)^2\right]^{\frac{3}{2}}}{12}$	½
		$\therefore \rho=145.50$	½
		--	
3.		**Solve any <u>THREE</u> of the following:**	**12**
	a)	Find the equation of the tangent to the curve $4x^2+9y^2=40$ at $(3,2)$	**04**
	Ans	$4x^2+9y^2=40$	
		$8x+18y\frac{dy}{dx}=0$	1
		$\therefore \frac{dy}{dx}=\frac{-8x}{18y}$	½
		at $(3,2)$	
		$\therefore \frac{dy}{dx}=\frac{-8(3)}{18(2)}$	
		$\therefore \frac{dy}{dx}=\frac{-2}{3}$	1
		$\therefore$ slope of tangent , $m=\frac{-2}{3}$	
		Equation of tangent at $(3,2)$ is	
		$y-2=\frac{-2}{3}(x-3)$	1
		$\therefore 3y-6=-2x+6$	
		$\therefore 2x+3y-12=0$	½
		--	

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	b)	Find $\frac{dy}{dx}$ if $y=\sec^{-1}\left[\frac{1}{4x^3-3x}\right]$	04
	Ans	$y=\sec^{-1}\left[\frac{1}{4x^3-3x}\right]$	
		Put $x=\cos\theta \Rightarrow \theta=\cos^{-1}x$	
		$\therefore y=\sec^{-1}\left[\frac{1}{4\cos^3\theta-3\cos\theta}\right]$	1
		$\therefore y=\sec^{-1}\left[\frac{1}{\cos 3\theta}\right]$	½
		$\therefore y=\sec^{-1}\left[\sec 3\theta\right]$	½
		$\therefore y=3\theta$	½
		$\therefore y=3\cos^{-1}x$	½
		$\therefore \frac{dy}{dx}=3\left(\frac{-1}{\sqrt{1-x^2}}\right)$	1
		$\therefore \frac{dy}{dx}=\frac{-3}{\sqrt{1-x^2}}$	
		---	
	c)	If $y^x=e^y$ prove that $\frac{dy}{dx}=\frac{(\log y)^2}{\log y-1}$	04
	Ans	$y^x=e^y$	
		taking log on both sides,	
		$\therefore \log y^x=\log e^y$	
		$\therefore x\log y=y\log e$	
		$\therefore x\log y=y$	1
		$\therefore x=\frac{y}{\log y}$	½
		diff.w.r.t.y	
		$\therefore \frac{dx}{dy}=\frac{\log y(1)-y\frac{1}{y}}{(\log y)^2}$	2
		$\therefore \frac{dx}{dy}=\frac{\log y-1}{(\log y)^2}$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	c)	$\therefore \frac{dy}{dx}=\frac{(\log y)^2}{\log y-1}$	½
		OR	
		$y^x=e^y$	
		taking log on both sides,	
		$\therefore \log y^x=\log e^y$	
		$\therefore x\log y=y\log e$	1
		$\therefore x\log y=y$ ------(i)	½
		diff.w.r.t.x	
		$\therefore x\frac{1}{y}\frac{dy}{dx}+\log y(1)=\frac{dy}{dx}$	1
		$\therefore \log y=\left(1-\frac{x}{y}\right)\frac{dy}{dx}$	
		$\therefore \frac{\log y}{\left(1-\frac{x}{y}\right)}=\frac{dy}{dx}$	½
		$\therefore \frac{\log y}{\left(1-\frac{1}{\log y}\right)}=\frac{dy}{dx}$ -----$(\because eq^n.(i))$	½
		$\therefore \frac{(\log y)^2}{(\log y-1)}=\frac{dy}{dx}$	
		$\therefore \frac{dy}{dx}=\frac{(\log y)^2}{\log y-1}$	½
		--	
	d)	Evaluate $\int\frac{(x-1)e^x}{x^2\sin^2\left(e^x/x\right)}\,dx$	**04**
	Ans	$\int\frac{(x-1)e^x}{x^2\sin^2\left(e^x/x\right)}\,dx$	
		Put $\frac{e^x}{x}=t$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	d)	$\therefore \frac{xe^x - e^x(1)}{x^2}dx = dt$	½
		$\therefore \frac{(x-1)e^x}{x^2}dx = dt$	
		$\therefore \int \frac{1}{\sin^2 t}\,dt$	½
		$= \int \operatorname{cosec}^2 t\,dt$	1
		$= -\cot t + c$	1
		$= -\cot\left(\frac{e^x}{x}\right) + c$	½
		--	
4.		**Solve any <u>THREE</u> of the following:**	**12**
	a)	Evaluate $\int \frac{dx}{4\cos^2 x + 9\sin^2 x}$	**04**
	Ans	$\int \frac{dx}{4\cos^2 x + 9\sin^2 x}$	
		$= \int \frac{\frac{dx}{\cos^2 x}}{\frac{4\cos^2 x + 9\sin^2 x}{\cos^2 x}}$	½
		$= \int \frac{\sec^2 x dx}{4 + 9\tan^2 x}$	½
		Put $\tan x = t$	½
		$\sec^2 x\,dx = dt$	
		$= \int \frac{dt}{4 + 9t^2}$	½
		$= \int \frac{dt}{(2)^2 + (3t)^2}$ or $= \frac{1}{9}\int \frac{dt}{\left(\frac{2}{3}\right)^2 + t^2}$	½
		$= \frac{1}{2}\frac{\tan^{-1}\left(\frac{3t}{2}\right)}{3} + c$ or $= \frac{1}{9\left(\frac{2}{3}\right)}\tan^{-1}\left(\frac{t}{\frac{2}{3}}\right) + c$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	a)	$=\frac{1}{6}\tan^{-1}\left(\frac{3\tan x}{2}\right)+c$	½
	b)	Evaluate: $\int \frac{\log x}{x[2+\log x][3+\log x]}\, dx$	**04**
	Ans	$\int \frac{\log x}{x[2+\log x][3+\log x]}\, dx$	
		Put $\log x = t$	½
		$\therefore \frac{1}{x}dx = dt$	
		$\int \frac{t}{(2+t)(3+t)}\, dt$	
		consider $\frac{t}{(2+t)(3+t)} = \frac{A}{2+t}+\frac{B}{3+t}$	½
		$\therefore t = A(3+t)+B(2+t)$	
		Put $t=-2$	½
		$A=-2$	
		Put $t=-3$	
		$B=3$	½
		$\therefore \frac{t}{(2+t)(3+t)} = \frac{-2}{2+t}+\frac{3}{3+t}$	½
		$\therefore \int \frac{t}{(2+t)(3+t)}\, dt = \int\left(\frac{-2}{2+t}+\frac{3}{3+t}\right) dt$	
		$=-2\log(2+t)+3\log(3+t)+c$	1
		$=-2\log(2+\log x)+3\log(3+\log x)+c$	½
	c)	Evaluate $\int_2^5 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\, dx$	**04**
	Ans	$I=\int_2^5 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\, dx$ --------------------(1)	
		$\therefore I=\int_2^5 \frac{\sqrt{(2+5-x)}}{\sqrt{7-(2+5-x)}+\sqrt{(2+5-x)}}\, dx$	1
		$\therefore I=\int_2^5 \frac{\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}}\, dx$--------------------(2)	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	c)	add (1) and (2)	
		$I+I=\int_2^5 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\,dx+\int_2^5 \frac{\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}}\,dx$	
		$\therefore 2I=\int_2^5 \frac{\sqrt{x}+\sqrt{7-x}}{\sqrt{7-x}+\sqrt{x}}\,dx$	½
		$\therefore 2I=\int_2^5 1\,dx$	
		$\therefore 2I=[x]_2^5$	1
		$\therefore 2I=5-2$	
		$\therefore 2I=3$	
		$\therefore I=\frac{3}{2}$	1
		--	
	d)	Evaluate $\int x.\tan^{-1}x\,dx$	**04**
	Ans	$\int \tan^{-1}x.xdx$	
		$=\tan^{-1}x\int xdx-\int\left(\int xdx\frac{d}{dx}\left(\tan^{-1}x\right)\right)dx$	½
		$=\tan^{-1}x\ \frac{x^2}{2}-\int\frac{x^2}{2}\frac{1}{1+x^2}dx$	1
		$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\frac{x^2}{1+x^2}dx$	
		$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\left(\frac{1+x^2-1}{1+x^2}\right)dx$	½
		$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\left(\frac{1+x^2}{1+x^2}-\frac{1}{1+x^2}\right)dx$	½
		$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\left(1-\frac{1}{1+x^2}\right)dx$	½
		$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\left(x-\tan^{-1}x\right)+c$	1
		--	
	e)	Evaluate $\int\frac{x}{(x+1)(x+2)}dx$	**04**

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	e)	Consider $\frac{x}{(x+1)(x+2)}=\frac{A}{x+1}+\frac{B}{x+2}$	½
		$\therefore x=A(x+2)+B(x+1)$	
		Put $x=-1$	
		$\therefore -1=A(-1+2)$	1
		$\therefore A=-1$	
		Put $x=-2$	
		$\therefore -2=B(-2+1)$	1
		$\therefore B=2$	
		$\frac{x}{(x+1)(x+2)}=\frac{-1}{x+1}+\frac{2}{x+2}$	
		$\therefore \int\frac{x}{(x+1)(x+2)}\,dx=-\int\frac{1}{x+1}dx+2\int\frac{1}{x+2}dx$	½
		$=-\log(x+1)+2\log(x+2)+c$	1

5.		**Solve any <u>TWO</u> of the following:**	**12**
	a)	Find by integration the area between the curves $y=x^2+1$ and line $y=2x+1$	**06**
	Ans	$y=x^2+1$ ---------(1)	
		$y=2x+1$	
		$\therefore \text{eq}^{n}.(1)\Rightarrow 2x+1=x^2+1$	
		$\therefore 2x+1-x^2-1=0$	
		$\therefore 2x-x^2=0$	
		$\therefore x=0,2$	1
		Area $A=\int_a^b(y_1-y_2)\,dx$	
		$\therefore A=\int_0^2\left(x^2+1-(2x+1)\right)dx$	1
		$\therefore A=\int_0^2\left(x^2-2x\right)dx$	
		$\therefore A=\left[\frac{x^3}{3}-x^2\right]_0^2$	2

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	a)	$\therefore A=\left[\frac{(2)^3}{3}-(2)^2\right]$	1
		$\therefore A=\frac{-4}{3}$	
		$\therefore \text{area } A=\frac{4}{3}$ or 1.333	1
	b)	Solve the following.	**06**
	(i)	Verify that $y=\log x$ is a solution of differential equation $x\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$	**03**
	Ans	$y=\log x$	
		$\frac{dy}{dx}=\frac{1}{x}$	1
		$\frac{d^2y}{dx^2}=-\frac{1}{x^2}$	1
		$L.H.S.=x\frac{d^2y}{dx^2}+\frac{dy}{dx}$	
		$=x\left(-\frac{1}{x^2}\right)+\frac{1}{x}$	½
		$=-\frac{1}{x}+\frac{1}{x}$	
		$=0$	½
		$=R.H.S.$	
		OR	
		$y=\log x$	
		$\therefore \frac{dy}{dx}=\frac{1}{x}$	1
		$\therefore x\frac{dy}{dx}=1$	½
		$\therefore x\frac{d^2y}{dx^2}+\frac{dy}{dx}(1)=0$	1
		$\therefore x\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	b)(ii)	The velocity of a particle is given by $v=t^2-6t+7$. Find distance covered in 3 second, initially $x=0$ when $t=0$	**03**
	Ans	$v=t^2-6t+7$	
		$\therefore \frac{dx}{dt}=t^2-6t+7$	½
		$\therefore dx=\left(t^2-6t+7\right)dt$	
		$\therefore \int dx=\int\left(t^2-6t+7\right)dt$	½
		$\therefore x=\frac{t^3}{3}-3t^2+7t+c$	1
		Initially $x=0$ when $t=0$	
		$\therefore c=0$	
		$\therefore x=\frac{t^3}{3}-3t^2+7t$	½
		Distance covered in 3 sec,	
		$\therefore x=\frac{(3)^3}{3}-3(3)^2+7(3)$	
		$\therefore x=3$	½
		--	
	c)	Solve the following.	**06**
	(i)	Solve $\left(1+x^2\right)dy-\left(1+y^2\right)dx=0$	**03**
	Ans	$\left(1+x^2\right)dy-\left(1+y^2\right)dx=0$	
		$\therefore\left(1+x^2\right)dy=\left(1+y^2\right)dx$ $\qquad \therefore \frac{dy}{1+y^2}=\frac{dx}{1+x^2}$	1
		$\therefore \int\frac{dy}{1+y^2}=\int\frac{dx}{1+x^2}$	1
		$\therefore \tan^{-1}y=\tan^{-1}x+c$	1
		--	
	c)(ii)	Solve $\frac{dy}{dx}+y\cot x=\cos x$	**03**
	Ans	$\frac{dy}{dx}+y\cot x=\cos x$	
		$\therefore$ Comparing with $\frac{dy}{dx}+Py=Q$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	c)(ii)	$P=\cot x\ ,\ \ Q=\cos x$	
		Integrating factor $IF=e^{\int \cot x dx}=e^{\log(\sin x)}=\sin x$	1
		$y.IF=\int Q.IFdx+c$	
		$\therefore y\sin x=\int \cos x.\sin x\ dx$	½
		$\therefore y\sin x=\frac{1}{2}\int 2\cos x.\sin x\ dx$	
		$\therefore y\sin x=\frac{1}{2}\int \sin 2x\ dx$	½
		$\therefore y\sin x=\frac{1}{2}\left(\frac{-\cos 2x}{2}\right)+c$	1
		$\therefore y\sin x=\frac{-\cos 2x}{4}+c$	
		OR	
		$\frac{dy}{dx}+y\cot x=\cos x$	
		$\therefore$ Comparing with $\frac{dy}{dx}+Py=Q$	
		$P=\cot x\ ,\ \ Q=\cos x$	
		Integrating factor $IF=e^{\int \cot x dx}=e^{\log(\sin x)}=\sin x$	1
		$y.IF=\int Q.IFdx+c$	
		$\therefore y\sin x=\int \cos x.\sin x\ dx$	
		Put $\sin x=t$	½
		$\therefore \cos x\ dx=dt$	
		$=\int t\ dt$	½
		$=\frac{t^2}{2}+c$	
		$\therefore y\sin x=\frac{(\sin x)^2}{2}+c$	1
		--	
6.		**Solve any TWO of the following:**	**12**
	a)	If $\omega_1=\frac{-1}{2}+i\frac{\sqrt{3}}{2}$, $\omega_2=\frac{-1}{2}-i\frac{\sqrt{3}}{2}$ show that $\omega_1 2=\omega_2$	**06**

WINTER– 18 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code: 22210**

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	a) Ans	$\omega_1^2 = \left(\frac{-1}{2}+i\frac{\sqrt{3}}{2}\right)^2$	
		$= \frac{1}{4} - 2\left(\frac{1}{2}\right)\left(i\frac{\sqrt{3}}{2}\right) + i^2\left(\frac{\sqrt{3}}{2}\right)^2$	2
		$= \frac{1}{4} - i\frac{\sqrt{3}}{2} - \frac{3}{4}$	
		$= \frac{-1}{2} - i\frac{\sqrt{3}}{2}$	2
		$= \omega_2$	2
		$\therefore \omega_1^2 = \omega_2$	
		--	
	b)	Find L$\left\{e^3 t\left(t^2+t\right)\right\}$	**06**
	Ans	L$\left\{e^3 t\left(t^2+t\right)\right\}$	
		$= \text{L}\left\{e^3\left(t^3+t^2\right)\right\}$	1
		$= e^3 \text{L}\left\{t^3+t^2\right\}$	1
		$= e^3\left\{L\left(t^3\right)+L\left(t^2\right)\right\}$	1
		$= e^3\left(\frac{3!}{s^{3+1}}+\frac{2!}{s^{2+1}}\right)$	2
		$= e^3\left(\frac{6}{s^4}+\frac{2}{s^3}\right)$	1
		--	
	c)	Find $L^{-1}\left\{\frac{2s^2-4}{(s+1)(s-2)(s-3)}\right\}$	**06**
	Ans	Let	
		$\frac{2s^2-4}{(s+1)(s-2)(s-3)} = \frac{A}{s+1}+\frac{B}{s-2}+\frac{C}{s-3}$	
		$2s^2-4 = (s-2)(s-3)A+(s+1)(s-3)B+(s+1)(s-2)C$	
		Put $s=-1$	
		$\therefore 2(-1)^2-4 = (-1-2)(-1-3)A$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	c)	$\therefore A=-\frac{1}{6}$	1
		Put $s=2$	
		$2(2)^2-4=(2+1)(2-3)B$	
		$\therefore B=\frac{-4}{3}$	1
		Put $s=3$	
		$2(3)^2-4=(3+1)(3-2)C$	
		$\therefore c=\frac{7}{2}$	1
		$\therefore \frac{2s^2-4}{(s+1)(s-2)(s-3)}=\frac{-\frac{1}{6}}{s+1}+\frac{\frac{-4}{3}}{s-2}+\frac{\frac{7}{2}}{s-3}$	
		$\therefore L^{-1}\left\{\frac{2s^2-4}{(s+1)(s-2)(s-3)}\right\}=-\frac{1}{6}L^{-1}\left\{\frac{1}{s+1}\right\}-\frac{4}{3}L^{-1}\left\{\frac{1}{s-2}\right\}+\frac{7}{2}L^{-1}\left\{\frac{1}{s-3}\right\}$	
		$=-\frac{1}{6}e^{-t}-\frac{4}{3}e^{2t}+\frac{7}{2}e^{3t}$	1+1+1
		--	
	d)	Solve differential equation using Laplace Transform.	**06**
		$\frac{dy}{dt}+2y=e^{-t}$, given $y(0)=2$	
	Ans	$\frac{dy}{dt}+2y=e^{-t}$	
		Apply Laplace Transform on both sides,	
		$\therefore L\left\{\frac{dy}{dt}+2y\right\}=L\left\{e^{-t}\right\}$	
		$\therefore sL(y)-y(0)+2L(y)=\frac{1}{s+1}$	1
		$\therefore sL(y)-2+2L(y)=\frac{1}{s+1}$	½
		$\therefore (s+2)L(y)-2=\frac{1}{s+1}$	½
		$\therefore (s+2)L(y)=\frac{1}{s+1}+2$	
		$\therefore (s+2)L(y)=\frac{2s+3}{s+1}$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	d)	$\therefore (s+2)L(y)=\dfrac{2s+3}{s+1}$	
		$\therefore L(y)=\dfrac{2s+3}{(s+1)(s+2)}$	½
		$\therefore y=L^{-1}\left\{\dfrac{2s+3}{(s+1)(s+2)}\right\}$	
		$\therefore \dfrac{2s+3}{(s+1)(s+2)}=\dfrac{A}{s+1}+\dfrac{B}{s+2}$	
		$\therefore 2s+3=A(s+2)+B(s+1)$	1
		Put $s=-1$	
		$\therefore A=1$	1
		Put $s=-2$	
		$\therefore B=1$	
		$\therefore \dfrac{2s+3}{(s+1)(s+2)}=\dfrac{1}{s+1}+\dfrac{1}{s+2}$	½
		$\therefore L^{-1}\left\{\dfrac{2s+3}{(s+1)(s+2)}\right\}=L^{-1}\left\{\dfrac{1}{s+1}+\dfrac{1}{s+2}\right\}$	
		$=L^{-1}\left\{\dfrac{1}{s+1}\right\}+L^{-1}\left\{\dfrac{1}{s+2}\right\}$	1
		$=e^{-t}+e^{-2t}$	
		--	
		Important Note	
		In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.	
		--	

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SUMMER– 19 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code: 22210**

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.		**Attempt any FIVE of the following:**	**10**
	a)	If $f(x)=3x^2-5x+7$ show that $f(-1)=3f(1)$	**02**
	Ans	$f(-1)=15$	1
		$f(1)=5$	
		$\therefore 3f(1)=15$	1
		$\therefore f(-1)=3f(1)$	
	b)	State whether the function $f(x)=3x^4+x^2+5-3\cos x+2\sin^2 x$ is even or odd	**02**
	Ans	$f(x)=3x^4+x^2+5-3\cos x+2\sin^2 x$	
		$\therefore f(-x)=3(-x)^4+(-x)^2+5-3\cos(-x)+2\sin^2(-x)$	½
		$\therefore f(-x)=3x^4+x^2+5-3\cos x+2\sin^2 x$	½
		$\therefore f(-x)=f(x)$	½
		$\therefore$ function is an even function	½
	c)	Find $\frac{dy}{dx}$ if $y=e^x.\sin^{-1}x$	**02**
	Ans	$y=e^x.\sin^{-1}x$	
		$\therefore \frac{dy}{dx}=e^x.\frac{1}{\sqrt{1-x^2}}+\sin^{-1}x.e^x$	1+1

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	c)	$\therefore \frac{dy}{dx} = e^x\left(\frac{1}{\sqrt{1-x^2}} + \sin^{-1} x\right)$	
	d)	Evaluate $e^{\int 2\log x\, dx}$	**02**
	Ans	$e^{\int 2\log x\, dx}$	
		$= e^{2\int \log x\, dx}$	
		$= e^{2\int \log x.1\, dx}$	½
		$= e^{2\left(\log x.x - \int x.\frac{1}{x}\, dx\right)}$	½
		$= e^{2\left(x\log x - \int 1\, dx\right)}$	½
		$= e^{2(x\log x - x)+c}$	½
		$= e^{2x(\log x-1)+c}$	
	e)	Evaluate $\int \sin^2 x\, dx$	**02**
	Ans	$\int \sin^2 x\, dx$	
		$= \frac{1}{2}\int 2\sin^2 x\, dx$	
		$= \frac{1}{2}\int (1-\cos 2x)\, dx$	1
		$= \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + c$	1
	f)	Find the area under the curve $y = x^2$ from $x = 0$ to $x = 3$ with $x-$axis.	**02**
	Ans	Area $A = \int_a^b y\, dx$	
		$= \int_0^3 x^2 dx$	½
		$= \left[\frac{x^3}{3}\right]_0^3$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	f)	$=\left(\frac{3^3}{3}-0\right)$	½
		$=9$	½
	g)	Express $z=1-i$ in Polar form.	**02**
	Ans	$z=1-i$	
		$\therefore x=1, y=-1$	
		$\therefore r=\lvert z\rvert=\sqrt{x^2+y^2}=\sqrt{(1)^2+(-1)^2}=\sqrt{2}$	½
		$\theta=2\pi-\tan^{-1}\left(\left\lvert\frac{y}{x}\right\rvert\right)=2\pi-\tan^{-1}\left(\left\lvert\frac{-1}{1}\right\rvert\right)=2\pi-\tan^{-1}(1)$	
		$\theta=2\pi-\frac{\pi}{4}=\frac{7\pi}{4}$	½
		$\therefore$ Polar form is $z=r(\cos\theta+i\sin\theta)$	
		$\therefore\ 1-i=\sqrt{2}\left(\cos\left(\frac{7\pi}{4}\right)+i\sin\left(\frac{7\pi}{4}\right)\right)$	1
2.		**Attempt any THREE of the following:**	**12**
	a)	Find $\frac{dy}{dx}$ if $x^2+y^2=4xy$	**04**
	Ans	$x^2+y^2=4xy$	
		$\therefore 2x+2y\frac{dy}{dx}=4\left(x\frac{dy}{dx}+y.1\right)$	2
		$\therefore 2x+2y\frac{dy}{dx}=4x\frac{dy}{dx}+4y$	
		$\therefore (2y-4x)\frac{dy}{dx}=4y-2x$	1
		$\therefore \frac{dy}{dx}=\frac{4y-2x}{2y-4x}$	1
		$\therefore \frac{dy}{dx}=\frac{2y-x}{y-2x}$	

Q. No.	Sub Q. N.	Answers	Marking Scheme
2.	b)	If $x=a\left(\theta+\sin\theta\right)$, $y=a\left(1-\cos\theta\right)$, find $\frac{dy}{dx}$ at $\theta=\frac{\pi}{2}$	**04**
	Ans	$x=a\left(\theta+\sin\theta\right)$ $\quad$ $y=a\left(1-\cos\theta\right)$	
		$\frac{dx}{d\theta}=a\left(1+\cos\theta\right)$ $\quad$ $\frac{dy}{d\theta}=a\left(0+\sin\theta\right)=a\sin\theta$	1+1
		$\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{a\sin\theta}{a\left(1+\cos\theta\right)}$	
		$\frac{dy}{dx}=\frac{\sin\theta}{1+\cos\theta}$	1
		at $\theta=\frac{\pi}{2}$	
		$\therefore\frac{dy}{dx}=\frac{\sin\frac{\pi}{2}}{1+\cos\frac{\pi}{2}}=1$	1
		--	
	c)	Find the radius of curvature of the curve $\sqrt{x}+\sqrt{y}=1$ at $\left(\frac{1}{4},\frac{1}{4}\right)$	**04**
	Ans	$\sqrt{x}+\sqrt{y}=1$	
		$\therefore\frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0$	½
		$\therefore\frac{dy}{dx}=-\frac{\sqrt{y}}{\sqrt{x}}$	½
		$\therefore\frac{d^2y}{dx^2}=-\frac{\left(\sqrt{x}\frac{1}{2\sqrt{y}}\frac{dy}{dx}-\sqrt{y}\frac{1}{2\sqrt{x}}\right)}{x}$	½
		$\therefore\frac{d^2y}{dx^2}=-\frac{\left(\frac{\sqrt{x}}{2\sqrt{y}}\cdot\frac{-\sqrt{y}}{\sqrt{x}}-\frac{\sqrt{y}}{2\sqrt{x}}\right)}{x}=-\frac{\left(\frac{-1}{2}-\frac{\sqrt{y}}{2\sqrt{x}}\right)}{x}$	½
		at $\left(\frac{1}{4},\frac{1}{4}\right)$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	d)	$\therefore \frac{dy}{dx} = -\frac{\sqrt{1/4}}{\sqrt{1/4}} = -1$	½
		$\therefore \frac{d^2y}{dx^2} = -\frac{\left(\frac{-1}{2} - \frac{\sqrt{1/4}}{2\sqrt{1/4}}\right)}{1/4} = 4$	½
		$\therefore$ Radius of curvature $= \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[1+(-1)^2\right]^{3/2}}{4}$	½
		$= 0.707$	½
		Find the maximum and minimum value of $x^3 - 9x^2 + 24y$	**04**
	d) Ans	Let $y = x^3 - 9x^2 + 24x$	
		$\therefore \frac{dy}{dx} = 3x^2 - 18x + 24$	½
		$\therefore \frac{d^2y}{dx^2} = 6x - 18$	½
		Consider $\frac{dy}{dx} = 0$	
		$3x^2 - 18x + 24 = 0$	
		$\therefore x = 2$ or $x = 4$	1
		at $x = 2$ $\therefore \frac{d^2y}{dx^2} = 6(2) - 18 = -6 < 0$	½
		$\therefore y$ is maximum at $x = 2$	
		$y_{\max} = (2)^3 - 9(2)^2 + 24(2) = 20$	½
		at $x = 4$ $\therefore \frac{d^2y}{dx^2} = 6(4) - 18 = 6 > 0$	½
		$\therefore y$ is minimum at $x = 4$	
		$y_{\min} = (4)^3 - 9(4)^2 + 24(4) = 16$	½
		***Note*: If students attempted to solve the question give appropriate marks.**	

SUMMER– 19 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code: 22210**

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.		**Attempt any <u>THREE</u> of the following:**	**12**
	a)	Find equation of tangent and normal to the curve $2x^2-xy+3y^2=18$ at $(3,1)$	**04**
	Ans	$2x^2-xy+3y^2=18$	
		$\therefore 4x-\left(x\frac{dy}{dx}+y.1\right)+6y\frac{dy}{dx}=0$	½
		$\therefore 4x-x\frac{dy}{dx}-y+6y\frac{dy}{dx}=0$	
		$\therefore (6y-x)\frac{dy}{dx}=y-4x$	
		$\therefore \frac{dy}{dx}=\frac{y-4x}{6y-x}$	½
		at $(3,1)$	
		$\therefore \frac{dy}{dx}=\frac{1-4(3)}{6(1)-3}$	
		$\therefore \frac{dy}{dx}=\frac{-11}{3}$	
		$\therefore$ slope of tangent , $m=\frac{-11}{3}$	½
		Equation of tangent at $(3,1)$ is	
		$y-1=\frac{-11}{3}(x-3)$	
		$\therefore 3y-3=-11x+33$	
		$\therefore 11x+3y-36=0$	1
		$\therefore$ slope of normal , $m'=\frac{-1}{m}=\frac{3}{11}$	½
		Equation of normal at $(3,1)$ is	
		$y-1=\frac{3}{11}(x-3)$	
		$\therefore 11y-11=3x-9$	
		$\therefore 3x-11y+2=0$	1
		--	

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	b)	Find $\frac{dy}{dx}$ if $y=x^x+(\sin x)^x$	04
	Ans	Let $u=x^x$	
		Taking log on both sides,	
		$\therefore \log u=\log x^x$	½
		$\therefore \log u = x\log x$	
		$\therefore \frac{1}{u}\frac{du}{dx}=x\frac{1}{x}+\log x.1$	½
		$\therefore \frac{1}{u}\frac{du}{dx}=1+\log x$	
		$\therefore \frac{du}{dx}=u(1+\log x)$	½
		Let $v=(\sin x)^x$	
		taking log on both sides,	
		$\therefore \log v=\log(\sin x)^x$	½
		$\therefore \log v = x\log\sin x$	
		$\therefore \frac{1}{v}\frac{dv}{dx}=x\frac{1}{\sin x}.\cos x+\log\sin x.1$	½
		$\therefore \frac{1}{v}\frac{dv}{dx}=x\cot x+\log\sin x$	
		$\therefore \frac{dv}{dx}=v(x\cot x+\log\sin x)$	½
		$\therefore \frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx}$	
		$\therefore \frac{dy}{dx}=x^x(1+\log x)+(\sin x)^x(x\cot x+\log\sin x)$	1
		--	
	c)	If $y=e^{3\sec x+4\tan x}$ find $\frac{dy}{dx}$	04
	Ans	$y=e^{3\sec x+4\tan x}$	
		$\therefore \frac{dy}{dx}=e^{3\sec x+4\tan x}(3\sec x\cdot\tan x+4\sec^2 x)$	4
		--	

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	d)	Evaluate $\int \frac{\sec^2 x}{(1+\tan x)(3+\tan x)}dx$	**04**
	Ans	$\int \frac{\sec^2 x}{(1+\tan x)(3+\tan x)}\, dx$	
		Put $\tan x = t$	
		$\therefore \sec^2 x dx = dt$	1
		$\therefore \int \frac{1}{(1+t)(3+t)}dt$	
		$\frac{1}{(1+t)(3+t)} = \frac{A}{1+t} + \frac{B}{3+t}$	½
		$\therefore 1 = A(3+t) + B(1+t)$	
		$\therefore$ Put $t=-1$, $A = \frac{1}{2}$	½
		Put $t=-3$, $B = \frac{-1}{2}$	½
		$\therefore \frac{1}{(1+t)(3+t)} = \frac{1/2}{1+t} - \frac{1/2}{3+t}$	
		$\therefore \int \frac{1}{(1+t)(3+t)}dt = \int \left(\frac{1/2}{1+t} - \frac{1/2}{3+t} \right) dt$	
		$= \frac{1}{2}\log(1+t) - \frac{1}{2}\log(3+t) + c$	1
		$= \frac{1}{2}\log(1+\tan x) - \frac{1}{2}\log(3+\tan x) + c$	½
		OR	
		$\int \frac{\sec^2 x}{(1+\tan x)(3+\tan x)}\, dx$	
		Put $\tan x = t$	
		$\therefore \sec^2 x dx = dt$	1
		$\int \frac{1}{(1+t)(3+t)}dt$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	d)	$=\int \frac{1}{t^2+4t+3}\,dt$	
		Third Term $=\frac{4^2}{4}=4$	½
		$=\int \frac{1}{t^2+4t+4-4+3}\,dt$	1
		$=\int \frac{1}{(t+2)^2-1}\,dt$	
		$=\frac{1}{2}\log\left\|\frac{t+2-1}{t+2+1}\right\|+c$	1
		$=\frac{1}{2}\log\left\|\frac{t+1}{t+3}\right\|+c$	
		$=\frac{1}{2}\log\left\|\frac{\tan x+1}{\tan x+3}\right\|+c$	½
4.		**Attempt any THREE of the following:**	**12**
	a)	Evaluate $\int x\tan^{-1}x\,dx$	**04**
	Ans	$\int \tan^{-1}x\cdot x dx$	
		$=\tan^{-1}x\int xdx-\int\left(\int xdx\frac{d}{dx}\left(\tan^{-1}x\right)\right)dx$	1
		$=\tan^{-1}x\ \frac{x^2}{2}-\int\frac{x^2}{2}\frac{1}{1+x^2}dx$	½
		$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\frac{x^2}{1+x^2}dx$	
		$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\left(\frac{1+x^2-1}{1+x^2}\right)dx$	½
		$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\left(\frac{1+x^2}{1+x^2}-\frac{1}{1+x^2}\right)dx$	½
		$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\left(1-\frac{1}{1+x^2}\right)dx$	½
		$=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\left(x-\tan^{-1}x\right)+c$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	b)	Evaluate $\int \frac{dx}{4+5\cos x}$	**04**
	Ans	$\int \frac{dx}{4+5\cos x}$	
		Put $\tan\frac{x}{2}=t$, $dx=\frac{2dt}{1+t^2}$, $\cos x=\frac{1-t^2}{1+t^2}$	1
		$=\int \frac{\frac{2dt}{1+t^2}}{4+5\left(\frac{1-t^2}{1+t^2}\right)}$	
		$=\int \frac{2dt}{4(1+t^2)+5(1-t^2)}$	½
		$=2\int \frac{dt}{4+4t^2+5-5t^2}$	
		$=2\int \frac{dt}{9-t^2}$	½
		$=2\int \frac{dt}{(3)^2-t^2}$	½
		$=2\frac{1}{2(3)}\log\left\|\frac{3+t}{3-t}\right\|+c$	1
		$=\frac{1}{3}\log\left\|\frac{3+\tan\frac{x}{2}}{3-\tan\frac{x}{2}}\right\|+c$	½
	c)	Evaluate $\int \frac{2x^2+5}{(x-1)(x+2)(x+3)}dx$	**04**
	Ans	$\int \frac{2x^2+5}{(x-1)(x+2)(x+3)}dx$	
		Consider $\frac{2x^2+5}{(x-1)(x+2)(x+3)}=\frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{x+3}$	½
		$\therefore 2x^2+5=A(x+2)(x+3)+B(x-1)(x+3)+C(x-1)(x+2)$	
		Put $x=1$	
		$\therefore 7=12A$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	c)	$\therefore A=\frac{7}{12}$	½
		Put $x=-2 \quad \Rightarrow 13=-3B$	
		$\therefore B=\frac{-13}{3}$	½
		Put $x=-3 \quad \Rightarrow 23=4C$	
		$\therefore C=\frac{23}{4}$	½
		$\therefore \int \frac{2x^2+5}{(x-1)(x+2)(x+3)}dx=\int\left(\frac{\frac{7}{12}}{x-1}+\frac{\frac{-13}{3}}{x+2}+\frac{\frac{23}{4}}{x+3}\right)dx$	½
		$=\frac{7}{12}\log(x-1)-\frac{13}{3}\log(x+2)+\frac{23}{4}\log(x+3)+c$	½+½+½
		---	
	d)	Evaluate $\int \frac{1}{\sqrt{16-6x-x^2}}\,dx$	**04**
	Ans	$\int \frac{1}{\sqrt{16-6x-x^2}}\,dx$	
		Third Term $=\frac{(6)^2}{4}=9$	1
		$=\int \frac{1}{\sqrt{16+9-9-6x-x^2}}\,dx$	1
		$=\int \frac{1}{\sqrt{25-(9+6x+x^2)}}\,dx$	
		$=\int \frac{1}{\sqrt{(5)^2-(x+3)^2}}\,dx$	1
		$=\sin^{-1}\left(\frac{x+3}{5}\right)+c$	1
		OR	
		$\int \frac{1}{\sqrt{16-6x-x^2}}\,dx$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	d)	$=\int \frac{1}{\sqrt{-\left(x^2+6x-16\right)}}\,dx$	½
		Third Term $=\frac{(6)^2}{4}=9$	1
		$=\int \frac{1}{\sqrt{-\left(x^2+6x+9-9-16\right)}}\,dx$	½
		$=\int \frac{1}{\sqrt{-\left(x^2+6x+9-25\right)}}\,dx$	
		$=\int \frac{1}{\sqrt{-\left[(x+3)^2-(5)^2\right]}}\,dx$	
		$=\int \frac{1}{\sqrt{(5)^2-(x+3)^2}}\,dx$	1
		$=\sin^{-1}\left(\frac{x+3}{5}\right)+c$	1
	e)	Evaluate $\int_0^{\pi/2} \frac{dx}{1+\cot x}$	**04**
	Ans	$\int_0^{\pi/2} \frac{dx}{1+\cot x}$	
		$\therefore I=\int_0^{\pi/2} \frac{1}{1+\frac{\cos x}{\sin x}}\,dx$	½
		$\therefore I=\int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}\,dx$ ---------------------(1)	
		$\therefore I=\int_0^{\pi/2} \frac{\sin\left(\pi/2-x\right)}{\sin\left(\pi/2-x\right)+\cos\left(\pi/2-x\right)}\,dx$	1
		$\therefore I=\int_0^{\pi/2} \frac{\cos x}{\cos x+\sin x}\,dx$ ---------------------(2)	
		add (1) and (2)	

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	e)	$\therefore I+I=\int_0^{\pi/2}\frac{\sin x}{\sin x+\cos x}\,dx+\int_0^{\pi/2}\frac{\cos x}{\cos x+\sin x}\,dx$	½
		$2I=\int_0^{\pi/2}\frac{\sin x+\cos x}{\sin x+\cos x}\,dx$	
		$2I=\int_0^{\pi/2}1\,dx$	
		$2I=\left[x\right]_0^{\pi/2}$	1
		$2I=\frac{\pi}{2}-0$	
		$\therefore I=\frac{\pi}{4}$	1
		--	
5.		**Attempt any <u>TWO</u> of the following:**	**04**
	a)	Find the area between the curves $y=x$ and $y=x^2$	
	Ans	$y=x$	
		$y=x^2$	
		$\therefore x-x^2=0$	
		$\therefore x(1-x)=0$	2
		$\therefore x=0,1$	1
		$\therefore A=\int_0^1\left(x-x^2\right)$	
		$A=\left(\frac{x^2}{2}-\frac{x^3}{3}\right)_0^1$	1
		$A=\left(\frac{1}{2}-\frac{1}{3}\right)$	1
		$\therefore A=\frac{1}{6}$ or 0.167	1
		--	
5.		Attempt the following	**12**
	b)(i)	Find the order and degree of the differential equation	

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	b) (i)	$\frac{d^2y}{dx^2}=\sqrt{1+\frac{dy}{dx}}$	**03**
	Ans	$\frac{d^2y}{dx^2}=\sqrt{1+\frac{dy}{dx}}$ Squaring both sides, we get $\left(\frac{d^2y}{dx^2}\right)^2=1+\frac{dy}{dx}$ $\therefore$ Order = 2 Degree = 2	1 1 1
	b)(ii)	Solve $\frac{dy}{dx}+y\cot x=\cos ecx$	**03**
	Ans	$\frac{dy}{dx}+y\cot x=\cos ecx$ Comparing with $\frac{dy}{dx}+Py=Q$ $P=\cot x$, $Q=\cos ecx$ Integrating factor $IF=e^{\int \cot x dx}=e^{\log(\sin x)}=\sin x$ $y.IF=\int Q.IFdx+c$ $\therefore y\sin x=\int \cos ecx.\sin x\ dx$ $\therefore y\sin x=\int 1\ dx$ $\therefore y\sin x=x+c$	1 1 ½ ½
	c)	If $L\frac{di}{dt}=30.\sin(10\pi t)$, find i in terms of t, given that $L=2$ and $i=0$ at $t=0$	**06**
	Ans	$Ldi=30\sin(10\pi t)dt$ $\int Ldi=\int 30\sin(10\pi t)dt$ $Li=30\left(\frac{-\cos(10\pi t)}{10\pi}\right)+c$	1 2

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	c)	$Li = \frac{-3\cos(10\pi t)}{\pi} + c$	
		at $t = 0, i = 0$	
		$L(0) = \frac{-3\cos(0)}{\pi} + c$	½
		$0 = \frac{-3}{\pi} + c$	
		$\therefore c = \frac{3}{\pi}$	1
		$\therefore Li = \frac{-3\cos(10\pi t)}{\pi} + \frac{3}{\pi}$	
		at $L = 2$	
		$\therefore 2i = \frac{-3\cos(10\pi t)}{\pi} + \frac{3}{\pi}$	½
		$\therefore i = \frac{3}{2\pi}(-\cos(10\pi t) + 1)$	1
		--	
6.		**Attempt any TWO of the following:**	**04**
	a)	Attempt the following	
	(i)	Express $\frac{2-\sqrt{3}i}{1+i}$ in $x+iy$ form.	**03**
	Ans	$\frac{2-\sqrt{3}i}{1+i}$	
		$= \frac{2-\sqrt{3}i}{1+i} \times \frac{1-i}{1-i}$	1
		$= \frac{2-2i-\sqrt{3}i+\sqrt{3}i^2}{1-i^2}$	½
		$= \frac{2-(2+\sqrt{3})i+\sqrt{3}(-1)}{1-i^2}$	
		$= \frac{2-(2+\sqrt{3})i-\sqrt{3}}{1+1}$	½

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	a)(i)	$=\dfrac{\left(2-\sqrt{3}\right)-\left(2+\sqrt{3}\right)i}{2}$	
		$=\dfrac{\left(2-\sqrt{3}\right)}{2}-\dfrac{\left(2+\sqrt{3}\right)i}{2}$	1
		--	
	a)(ii)	Find $L\left\{e^{-4t}t^2\right\}$	**03**
	Ans	$L\left\{e^{-4t}t^2\right\}$	
		$L\left\{t^2\right\}=\dfrac{2!}{s^3}=\dfrac{2}{s^3}$	1
		$\therefore L\left\{e^{-4t}t^2\right\}=\dfrac{2}{\left(s+4\right)^3}$	2
		--	
	b)	Find $L^{-1}\left\{\dfrac{2s^2-4}{\left(s+1\right)\left(s-2\right)\left(s-3\right)}\right\}$	**04**
	Ans	$L^{-1}\left\{\dfrac{2s^2-4}{\left(s+1\right)\left(s-2\right)\left(s-3\right)}\right\}$	
		Let	
		$\dfrac{2s^2-4}{\left(s+1\right)\left(s-2\right)\left(s-3\right)}=\dfrac{A}{s+1}+\dfrac{B}{s-2}+\dfrac{C}{s-3}$	
		$2s^2-4=\left(s-2\right)\left(s-3\right)A+\left(s+1\right)\left(s-3\right)B+\left(s+1\right)\left(s-2\right)C$	
		Put $s=-1$	
		$\therefore -2=12A$	
		$\therefore A=-\dfrac{1}{6}$	1
		Put $s=2$	
		$4=-3B$	
		$\therefore B=-\dfrac{4}{3}$	1
		Put $s=3$	
		$14=4C$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	b)	$\therefore C=\frac{7}{2}$	1
		$\therefore \frac{2s^2-4}{(s+1)(s-2)(s-3)}=\frac{-\frac{1}{6}}{s+1}+\frac{-\frac{4}{3}}{s-2}+\frac{\frac{7}{2}}{s-3}$	
		$\therefore L^{-1}\left\{\frac{2s^2-4}{(s+1)(s-2)(s-3)}\right\}=-\frac{1}{6}L^{-1}\left\{\frac{1}{s+1}\right\}-\frac{4}{3}L^{-1}\left\{\frac{1}{s-2}\right\}+\frac{7}{2}L^{-1}\left\{\frac{1}{s-3}\right\}$	
		$=-\frac{1}{6}e^{-t}-\frac{4}{3}e^{2t}+\frac{7}{2}e^{3t}$	1+1+1
		--	
	c)	Solve using Laplace transform	**06**
		$\frac{dx}{dt}+2x=e^{-t}$, given that $x(0)=2$	
	Ans	$\frac{dx}{dt}+2x=e^{-t}$	
		$\therefore L\left\{\frac{dx}{dt}+2x\right\}=L\left\{e^{-t}\right\}$	
		$\therefore sL(x)-x(0)+2L(x)=\frac{1}{s+1}$	1
		$\therefore sL(x)-2+2L(x)=\frac{1}{s+1}$	
		$\therefore (s+2)L(x)-2=\frac{1}{s+1}$	
		$\therefore (s+2)L(x)=\frac{1}{s+1}+2$	
		$\therefore (s+2)L(x)=\frac{2s+3}{s+1}$	
		$\therefore L(x)=\frac{2s+3}{(s+1)(s+2)}$	1
		$\therefore x=L^{-1}\left\{\frac{2s+3}{(s+1)(s+2)}\right\}$	½
		$\therefore \frac{2s+3}{(s+1)(s+2)}=\frac{A}{s+1}+\frac{B}{s+2}$	½
		$\therefore 2s+3=A(s+2)+B(s+1)$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	c)	Put $s=-1$	
		$\therefore A=1$	½
		Put $s=-2$	
		$\therefore B=1$	½
		$\therefore \frac{2s+3}{(s+1)(s+2)}=\frac{1}{s+1}+\frac{1}{s+2}$	
		$\therefore L^{-1}\left\{\frac{2s+3}{(s+1)(s+2)}\right\}=L^{-1}\left\{\frac{1}{s+1}+\frac{1}{s+2}\right\}$	
		$=L^{-1}\left\{\frac{1}{s+1}\right\}+L^{-1}\left\{\frac{1}{s+2}\right\}$	
		$=e^{-t}+e^{-2t}$	1+1

Important Note

In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.

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WINTER– 19 EXAMINATION

Subject Name: Applied Mathematics **Model Answer** **Subject Code: 22210**

Important Instructions to Examiners:

1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
3) The language errors such as grammatical, spelling errors should not be given more Importance (Not applicable for subject English and Communication Skills.
4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answers and model answer.
6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.		**Attempt any FIVE of the following:**	**10**
	a)	If $f\left(x\right)=\tan x$, show that $f\left(2x\right)=\frac{2f\left(x\right)}{1-\left[f\left(x\right)\right]^2}$	**02**
	Ans	$f\left(2x\right)$	
		$=\tan 2x$	½
		$=\frac{2\tan x}{1-\tan^2 x}$	1
		$=\frac{2f\left(x\right)}{1-\left[f\left(x\right)\right]^2}$	½
		OR	
		$\frac{2f\left(x\right)}{1-\left[f\left(x\right)\right]^2}$	
		$=\frac{2\tan x}{1-\tan^2 x}$	½
		$=\tan 2x$	1
		$=f\left(2x\right)$	½
		--	
	b)	State whether the function $f(x)=\frac{e^x+e^{-x}}{2}$ is even or odd.	**02**

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Subject Name: Applied Mathematics **Model Answer** **Subject Code: 22210**

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	b) Ans	$f(x)=\dfrac{e^{x}+e^{-x}}{2}$	
		$\therefore f(-x)=\dfrac{e^{-x}+e^{-(-x)}}{2}$	½
		$\therefore f(-x)=\dfrac{e^{-x}+e^{x}}{2}$	½
		$\therefore f(-x)=f(x)$	½
		$\therefore$ function is even.	½
		--	
	c) Ans	Find $\dfrac{dy}{dx}$ if $y=x.e^{x}$	**02**
		$y=x.e^{x}$	
		$\therefore \dfrac{dy}{dx}=x\dfrac{d}{dx}\left(e^{x}\right)+e^{x}\dfrac{d}{dx}(x)$	
		$\therefore \dfrac{dy}{dx}=xe^{x}+e^{x}.1$	2
		$\therefore \dfrac{dy}{dx}=xe^{x}+e^{x}$	
		--	
	d) Ans	Evaluate $\int \tan^{-1}x\,dx$	**02**
		$\int \tan^{-1}x\,dx=\int \tan^{-1}x\,.1\,dx$	
		$=\tan^{-1}x\int 1dx-\int\left(\left(\int 1dx\right)\dfrac{d}{dx}\left(\tan^{-1}x\right)\right)dx$	½
		$=x\tan^{-1}x-\int x\dfrac{1}{1+x^{2}}dx$	½
		$=x\tan^{-1}x-\int \dfrac{x}{1+x^{2}}dx$	
		$=x\tan^{-1}x-\dfrac{1}{2}\int \dfrac{2x}{1+x^{2}}dx$	½
		$=x\tan^{-1}x-\dfrac{1}{2}\log\left(1+x^{2}\right)+c$	½
		--	
	e)	Evaluate $\int \sqrt{1+\sin 2x}\,dx$	**02**

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Subject Name: Applied Mathematics **Model Answer** **Subject Code: 22210**

Q. No.	Sub Q.N.	Answers	Marking Scheme
1.	e)	$\int \sqrt{1+\sin 2x}\ dx$	
	Ans	$= \int \sqrt{\sin^2 x+\cos^2 x+2\sin x\cos x}\ dx$	½
		$= \int \sqrt{(\sin x+\cos x)^2}\ dx$	½
		$= \int (\sin x+\cos x)\ dx$	½
		$= -\cos x+\sin x+c$	½
	f)	Find the area bounded by the curve $y=\sin x$ and the x-axis from $x=0$ to $x=\pi$	**02**
	Ans	Area $A=\int_a^b y\ dx$	
		$= \int_0^{\pi} \sin x\ dx$	½
		$= [-\cos x]_0^{\pi}$	½
		$= [-\cos\pi]-[-\cos 0]$	½
		$= -(-1)-(-1)$	
		$= 2$	½
	g)	Express in the form $a+ib$, $Z=\dfrac{1+i}{2-i}$, where $a,b, \in$ IR . $i=\sqrt{-1}$	**02**
	Ans	$Z=\dfrac{1+i}{2-i}$	
		$=\dfrac{1+i}{2-i}\times\dfrac{2+i}{2+i}$	½
		$=\dfrac{2+i+2i+i^2}{2^2-i^2}$	½
		$=\dfrac{2+3i-1}{4+1}$	½
		$=\dfrac{1+3i}{5}$	
		$=\dfrac{1}{5}+\dfrac{3i}{5}$	½

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Subject Name: Applied Mathematics **Model Answer** **Subject Code: 22210**

Q. No.	Sub Q. N.	Answers	Marking Scheme
2.		**Attempt any THREE of the following:**	**12**
	a)	If $x = a(\theta - \sin\theta)$, $y = a(1-\cos\theta)$ find $\frac{dy}{dx}$	**04**
	Ans	$x = a(\theta - \sin\theta)$, $y = a(1-\cos\theta)$	
		$x = a(\theta - \sin\theta)$	
		$\therefore \frac{dx}{d\theta} = a(1-\cos\theta)$	1
		$y = a(1-\cos\theta)$	
		$\therefore \frac{dy}{d\theta} = a(0+\sin\theta) = a\sin\theta$	1
		$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\sin\theta}{a(1-\cos\theta)}$	1
		$\frac{dy}{dx} = \frac{\sin\theta}{1-\cos\theta}$	1
		--	
	b)	If $x^2 + y^2 = xy$ find $\frac{dy}{dx}$	**04**
	Ans	$x^2 + y^2 = xy$	
		$\therefore 2x + 2y\frac{dy}{dx} = x\frac{dy}{dx} + y.1$	1
		$\therefore 2x + 2y\frac{dy}{dx} = x\frac{dy}{dx} + y$	1
		$\therefore (2y - x)\frac{dy}{dx} = y - 2x$	1
		$\therefore \frac{dy}{dx} = \frac{y-2x}{2y-x}$	1
		--	
	c)	A metal wire 36 cm long is bent to form a rectangle. Find its dimensions when its area is maximum.	**04**
	Ans	Let length of rectangle = x , breadth = y	
		$\therefore 2x + 2y = 36$	
		$\therefore y = 18 - x$	1
		Area $A = x \times y$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	c)	$A = x(18-x)$	
		$\therefore A = 18x - x^2$	1
		$\therefore \frac{dA}{dx} = 18 - 2x$	½
		$\therefore \frac{d^2A}{dx^2} = -2$	½
		Let $\frac{dA}{dx} = 0$	
		$\therefore 18 - 2x = 0$	
		$\therefore x = 9$	½
		at $x = 9$	
		$\frac{d^2A}{dx^2} = -2 < 0$	
		Area is maximum at $x = 9$	½
		Length $= 9$; breadth $= 9$	
	d)	A beam is bent in the form of the curve $y = 2\sin x - \sin 2x$. Find the radius of curvature of the beam at this point at $x = \frac{\pi}{2}$	**04**
	Ans	$y = 2\sin x - \sin 2x$	
		$\therefore \frac{dy}{dx} = 2\cos x - 2\cos 2x$	½
		$\therefore \frac{d^2y}{dx^2} = -2\sin x + 4\sin 2x$	½
		at $x = \frac{\pi}{2}$	
		$\therefore \frac{dy}{dx} = 2\cos\left(\frac{\pi}{2}\right) - 2\cos 2\left(\frac{\pi}{2}\right) = 2$	½
		$\therefore \frac{d^2y}{dx^2} = -2\sin\left(\frac{\pi}{2}\right) + 4\sin 2\left(\frac{\pi}{2}\right) = -2$	½
		$\therefore$ Radius of curvature $= \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[1+(2)^2\right]^{3/2}}{-2}$	1

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Subject Name: Applied Mathematics **Model Answer** **Subject Code: 22210**

Q. No.	Sub Q.N.	Answers	Marking Scheme
2.	d)	$\therefore$ Radius of curvature $= -5.59$ $= 5.59$	1
		--	
3.		**Attempt any THREE of the following:**	**12**
	a)	Find the equation of the tangent and normal to the curve $4x^2+9y^2=40$ at $(1,2)$	**04**
	Ans	$4x^2+9y^2=40$	
		$\therefore 8x+18y\frac{dy}{dx}=0$	½
		$\therefore \frac{dy}{dx}=\frac{-8x}{18y}$	
		$\therefore \frac{dy}{dx}=\frac{-4x}{9y}$	½
		at $(1,2)$	
		$\therefore \frac{dy}{dx}=\frac{-4(1)}{9(2)}$	
		$\therefore \frac{dy}{dx}=\frac{-2}{9}$	
		$\therefore$ slope of tangent , $m=\frac{-2}{9}$	½
		Equation of tangent at $(1,2)$ is	
		$y-2=\frac{-2}{9}(x-1)$	½
		$\therefore 9y-18=-2x+2$	
		$\therefore 2x+9y-20=0$	½
		$\therefore$ slope of normal , $m'=\frac{-1}{m}=\frac{9}{2}$	½
		Equation of normal at $(1,2)$ is	
		$y-2=\frac{9}{2}(x-1)$	½
		$\therefore 2y-4=9x-9$	
		$\therefore 9x-2y-5=0$	½
		--	

WINTER– 19 EXAMINATION

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	b)	Find $\frac{dy}{dx}$ if $y=x^{\sin x}+\left(\tan x\right)^x$	**04**
	Ans	Let $u=x^{\sin x}$	
		$\therefore \log u=\log x^{\sin x}$	
		$\therefore \log u=\sin x\log x$	½
		$\therefore \frac{1}{u}\frac{du}{dx}=\sin x\cdot\frac{1}{x}+\log x.\cos x$	1
		$\therefore \frac{1}{u}\frac{du}{dx}=\frac{\sin x}{x}+\cos x\log x$	
		$\therefore \frac{du}{dx}=u\left(\frac{\sin x}{x}+\cos x\log x\right)$	
		$\therefore \frac{du}{dx}=x^{\sin x}\left(\frac{\sin x}{x}+\cos x\log x\right)$	½
		Let $v=\left(\tan x\right)^x$	
		$\therefore \log v=\log\left(\tan x\right)^x$	
		$\therefore \log v=x\log\left(\tan x\right)$	½
		$\therefore \frac{1}{v}\frac{dv}{dx}=x\cdot\frac{1}{\tan x}.\sec^2 x+\log\left(\tan x\right)\cdot 1$	1
		$\therefore \frac{1}{v}\frac{dv}{dx}=\frac{x\sec^2 x}{\tan x}+\log\left(\tan x\right)$	
		$\therefore \frac{dv}{dx}=v\left[\frac{x\sec^2 x}{\tan x}+\log\left(\tan x\right)\right]$	
		$\therefore \frac{dv}{dx}=\left(\tan x\right)^x\left[\frac{x\sec^2 x}{\tan x}+\log\left(\tan x\right)\right]$	½
		$\therefore \frac{dy}{dx}=x^{\sin x}\left(\frac{\sin x}{x}+\cos x\log x\right)+\left(\tan x\right)^x\left[\frac{x\sec^2 x}{\tan x}+\log\left(\tan x\right)\right]$	
		---	
	c)	Find $\frac{dy}{dx}$ if $y=\log\left[x+\sqrt{x^2+a^2}\right]$	**04**
	Ans	$y=\log\left[x+\sqrt{x^2+a^2}\right]$	
		$\therefore \frac{dy}{dx}=\frac{1}{x+\sqrt{x^2+a^2}}\left(1+\frac{1}{2\sqrt{x^2+a^2}}\left(2x+0\right)\right)$	2

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Subject Name: Applied Mathematics **Model Answer** **Subject Code: 22210**

Q. No.	Sub Q.N.	Answers	Marking Scheme
3.	c)	$\therefore \dfrac{dy}{dx}=\dfrac{1}{x+\sqrt{x^2+a^2}}\left(1+\dfrac{x}{\sqrt{x^2+a^2}}\right)$	
		$\therefore \dfrac{dy}{dx}=\dfrac{1}{x+\sqrt{x^2+a^2}}\left(\dfrac{x+\sqrt{x^2+a^2}}{\sqrt{x^2+a^2}}\right)$	
		$\therefore \dfrac{dy}{dx}=\dfrac{1}{\sqrt{x^2+a^2}}$	2
		--	
	d)	Evaluate $\int \dfrac{dx}{4+5\cos x}$	**04**
	Ans	$\int \dfrac{dx}{4+5\cos x}$	
		Put $\tan\dfrac{x}{2}=t$, $dx=\dfrac{2dt}{1+t^2}$	
		$\cos x=\dfrac{1-t^2}{1+t^2}$	
		$\int \dfrac{\dfrac{2dt}{1+t^2}}{4+5\left(\dfrac{1-t^2}{1+t^2}\right)}$	1
		$=\int \dfrac{2dt}{4\left(1+t^2\right)+5\left(1-t^2\right)}$	½
		$=2\int \dfrac{dt}{4+4t^2+5-5t^2}$	
		$=2\int \dfrac{dt}{9-t^2}$	1
		$=2\int \dfrac{dt}{3^2-t^2}$	
		$=2\dfrac{1}{2\cdot 3}\log\left\|\dfrac{3+t}{3-t}\right\|+c$	1
		$=\dfrac{1}{3}\log\left\|\dfrac{3+\tan\dfrac{x}{2}}{3-\tan\dfrac{x}{2}}\right\|+c$	½
		--	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
4.		**Attempt any THREE of the following:**	**12**
	a)	Evaluate $\int \frac{(x-1)e^x}{x^2 \sin^2\left(\frac{e^x}{x}\right)} dx$	**04**
	Ans	$\int \frac{(x-1)e^x}{x^2 \sin^2\left(\frac{e^x}{x}\right)} dx$	
		Put $\frac{e^x}{x} = t$	
		$\therefore \frac{xe^x - e^x(1)}{x^2} dx = dt$	1
		$\therefore \frac{(x-1)e^x}{x^2} dx = dt$	
		$\therefore \int \frac{1}{\sin^2 t} dt$	½
		$= \int \cos ec^2 t \ dt$	1
		$= -\cot t + c$	1
		$= -\cot\left(\frac{e^x}{x}\right) + c$	½
		--	
	b)	Evaluate $\int \sin^3 x \ dx$	**04**
	Ans	$\int \sin^3 x \ dx$	
		since $\sin 3x = 3\sin x - 4\sin^3 x$ $\quad \therefore \sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)$	
		$\therefore \int \frac{1}{4}(3\sin x - \sin 3x) \ dx$	2
		$= \frac{1}{4}\left(-3\cos x + \frac{\cos 3x}{3}\right) + c$	2
		OR $\int \sin^3 x \ dx$	½
		$= \int \sin^2 x \ \sin x \ dx$	
		$= \int (1-\cos^2 x) \sin x \ dx$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	b)	Put $\cos x = t$ $\therefore -\sin x dx = dt$ $\therefore \sin x dx = -dt$	1
		$\therefore \int (1-t^2)(-dt)$ $= -\int (1-t^2) dt$ $= -\left(t - \frac{t^3}{3}\right) + c$	1
		$= -\left(\cos x - \frac{\cos^3 x}{3}\right) + c$	½
		--	
	c)	Evaluate $\int \frac{2x^2+5}{(x-1)(x+2)(x+3)} dx$	**04**
	Ans	$\int \frac{2x^2+5}{(x-1)(x+2)(x+3)} dx$	
		Consider $\frac{2x^2+5}{(x-1)(x+2)(x+3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x+3}$	½
		$\therefore 2x^2+5 = A(x+2)(x+3) + B(x-1)(x+3) + C(x-1)(x+2)$ Put $x = 1$ $\therefore 7 = 12A$ $\therefore A = \frac{7}{12}$	½
		Put $x = -2$ $\Rightarrow 13 = -3B$ $\therefore B = \frac{-13}{3}$	½
		Put $x = -3$ $\Rightarrow 23 = 4C$ $\therefore C = \frac{23}{4}$	½
		$\therefore \int \frac{2x^2+5}{(x-1)(x+2)(x+3)} dx = \int \left(\frac{\frac{7}{12}}{x-1} + \frac{\frac{-13}{3}}{x+2} + \frac{\frac{23}{4}}{x+3} \right) dx$ $= \frac{7}{12}\log(x-1) - \frac{13}{3}\log(x+2) + \frac{23}{4}\log(x+3) + c$	2

Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	d)	Evaluate $\int x^2.e^{3x}\ dx$	**04**
	Ans	$\int x^2.e^{3x}\ dx$	
		$= x^2\left(\int e^{3x}dx\right) - \int\left(\int e^{3x}dx.\frac{d}{dx}\left(x^2\right)\right)dx$	1
		$= x^2\frac{e^{3x}}{3} - \int\frac{e^{3x}}{3}.2x\ dx$	1
		$= \frac{x^2e^{3x}}{3} - \frac{2}{3}\int xe^{3x}\ dx$	
		$= \frac{x^2e^{3x}}{3} - \frac{2}{3}\left[x\left(\int e^{3x}dx\right) - \int\left(\int e^{3x}dx.\frac{d}{dx}(x)\right)dx\right]$	½
		$= \frac{x^2e^{3x}}{3} - \frac{2}{3}\left[x\frac{e^{3x}}{3} - \int\frac{e^{3x}}{3}.1\ dx\right]$	1
		$= \frac{x^2e^{3x}}{3} - \frac{2}{3}\left[\frac{xe^{3x}}{3} - \frac{e^{3x}}{9}\right] + c$	½
		--	
	e)	Evaluate $\int_0^5 \frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}\ dx$	**04**
	Ans	$I = \int_0^5 \frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}\ dx$ --------------------(1)	
		$I = \int_0^5 \frac{\sqrt{5-(5-x)}}{\sqrt{5-x}+\sqrt{5-(5-x)}}\ dx$	1
		$\therefore I = \int_0^5 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\ dx$ --------------------(2)	
		add (1) and (2)	
		$I + I = \int_0^5 \frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}\ dx + \int_0^5 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\ dx$	1
		$2I = \int_0^5 \frac{\sqrt{5-x}+\sqrt{x}}{\sqrt{x}+\sqrt{5-x}}\ dx$	
		$2I = \int_0^5 1\ dx$	

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Q. No.	Sub Q.N.	Answers	Marking Scheme
4.	e)	$2I=\left[x\right]_0^5$	1
		$2I=5-0$	
		$I=\dfrac{5}{2}$	1
		--	
5.		**Attempt any <u>TWO</u> of the following:**	**12**
	a)	Find the area of the circle $x^2+y^2=36$ by using definite integration.	**06**
	Ans	$x^2+y^2=36$	
		$\therefore y^2=36-x^2$	
		$\therefore y=\sqrt{36-x^2}$	1
		$A=4\int_0^6\sqrt{36-x^2}\,dx$	
		$=4\int_0^6\sqrt{6^2-x^2}\,dx$	
		$=4\left[\frac{x}{2}\sqrt{6^2-x^2}+\frac{6^2}{2}\sin^{-1}\left(\frac{x}{6}\right)\right]_0^6$	2
		$=4\left[0+\frac{36}{2}\sin^{-1}(1)-0\right]$	1
		$=4\left[\frac{36}{2}\cdot\frac{\pi}{2}\right]$	1
		$=36\pi$	1
		--	
	b) i)	Find the order and degree of D.E.	**03**
		$\sqrt{\frac{d^2y}{dx^2}}-\frac{dy}{dx}-xy^2=0$	
	Ans	$\sqrt{\frac{d^2y}{dx^2}}=\frac{dy}{dx}+xy^2$	
		$\therefore \frac{d^2y}{dx^2}=\left(\frac{dy}{dx}+xy^2\right)^2$	1
		$\therefore$ Order $=2$	1
		Degree $=1$	1

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	b) ii)	Solve D.E. $x\cdot\frac{dy}{dx}+y=x^3$	**03**
	Ans	$x\cdot\frac{dy}{dx}+y=x^3$	
		$\therefore \frac{dy}{dx}+\frac{y}{x}=x^2$	
		Comparing with $\frac{dy}{dx}+Py=Q$	
		$\therefore P=\frac{1}{x},\ Q=x^2$	
		$\therefore$ Integrating Factor $=e^{\int Pdx}$	
		$=e^{\int \frac{1}{x}dx}$	
		$=e^{\log x}$	
		$=x$	1
		$\therefore$ Solution is,	
		$y\cdot \text{I.F.}=\int Q\cdot \text{I.F.}dx+c$	
		$\therefore y\cdot x=\int x^2\cdot x dx+c$	1
		$\therefore xy=\int x^3 dx+c$	
		$\therefore xy=\frac{x^4}{4}+c$	1
		--	

Q. No.	Sub Q.N.	Answers	Marking Scheme
5.	c)	The velocity of a particle is given by $v = t^2 - 6t + 7$. Find distance covered in 3 seconds.	**06**
	Ans	$v = t^2 - 6t + 7$	
		$\therefore v = \frac{dx}{dt} = t^2 - 6t + 7$	1
		$\therefore dx = \left(t^2 - 6t + 7\right)dt$	
		$\therefore \int dx = \int \left(t^2 - 6t + 7\right)dt$	1
		$\therefore x = \frac{t^3}{3} - 3t^2 + 7t + c$	1
		at $t = 0$, $x = 0$ $\therefore c = 0$	1
		$\therefore x = \frac{t^3}{3} - 3t^2 + 7t$	1
		at $t = 3$	
		$\therefore x = \frac{(3)^3}{3} - 3(3)^2 + 7(3)$	
		$\therefore x = 3$	1
6.		**Attempt any <u>TWO</u> of the following:**	**12**
	a) (i)	Express in polar form, $Z = 1 + i\sqrt{3}$	**03**
	Ans	$Z = 1 + i\sqrt{3}$	
		Comparing with $Z = x + iy$	
		$\therefore x = 1, y = \sqrt{3}$	
		$r = \sqrt{x^2 + y^2}$	
		$r = \sqrt{(1)^2 + \left(\sqrt{3}\right)^2} = 2$	1
		$\theta = \tan^{-1}\left\|\frac{y}{x}\right\|$	
		$\theta = \tan^{-1}\left\|\frac{\sqrt{3}}{1}\right\|$	
		$\theta = \tan^{-1}\sqrt{3}$	
		$\theta = 60^0$ or $\frac{\pi}{3}$	1

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Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	a) (i)	In polar form, $Z = r(\cos\theta + i\sin\theta)$ $Z = 2(\cos 60^0 + i\sin 60^0)$ or $Z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$	1
		--	
	a)(ii)	Find $L\{\sin 3t + \cos 2t\}$	**03**
	Ans	$L\{\sin 3t + \cos 2t\}$ $= \frac{3}{s^2+3^2} + \frac{s}{s^2+2^2}$ $= \frac{3}{s^2+9} + \frac{s}{s^2+4}$	 2 1
		--	
	b)	Find $L^{-1}\left\{\frac{2s+3}{(s+2)(s+6)}\right\}$	**06**
	Ans	$L^{-1}\left\{\frac{2s+3}{(s+2)(s+6)}\right\}$ Let $\frac{2s+3}{(s+2)(s+6)} = \frac{A}{s+2} + \frac{B}{s+6}$ $\therefore 2s+3 = (s+6)A + (s+2)B$ Put $s = -2$ $\therefore -1 = 4A$ $\therefore A = -\frac{1}{4}$ Put $s = -6$ $-9 = -4B$ $\therefore B = \frac{9}{4}$ $\therefore \frac{2s+3}{(s+2)(s+6)} = \frac{\frac{-1}{4}}{s+2} + \frac{\frac{9}{4}}{s+6}$	 ½ 1 1 ½

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	b)	$\therefore L^{-1}\left\{\frac{2s+3}{(s+2)(s+6)}\right\}=-\frac{1}{4}L^{-1}\left\{\frac{1}{s+2}\right\}+\frac{9}{4}L^{-1}\left\{\frac{1}{s+6}\right\}$	1
		$=-\frac{1}{4}e^{-2t}+\frac{9}{4}e^{-6t}$	2
		---	
	c)	Solve the differential equation using Laplace Transformation.	**06**
		$\frac{dy}{dt}-3y=t\cdot e^{-2t}\ ,\ y(0)=0$	
	Ans	$\frac{dy}{dt}-3y=t\cdot e^{-2t}$	
		$\therefore L\left\{\frac{dy}{dt}-3y\right\}=L\left\{t\cdot e^{-2t}\right\}$	
		$\therefore sL(y)-y(0)-3L(y)=(-1)^1\frac{d}{ds}\left(\frac{1}{s+2}\right)$	1
		$\therefore sL(y)-0-3L(y)=-1\cdot\frac{-1}{(s+2)^2}$	½
		$\therefore (s-3)L(y)=\frac{1}{(s+2)^2}$	
		$\therefore L(y)=\frac{1}{(s+2)^2(s-3)}$	1
		$\therefore y=L^{-1}\left\{\frac{1}{(s+2)^2(s-3)}\right\}$	
		Let $\frac{1}{(s+2)^2(s-3)}=\frac{A}{s+2}+\frac{B}{(s+2)^2}+\frac{C}{s-3}$	
		$\therefore 1=A(s+2)(s-3)+B(s-3)+C(s+2)^2$	
		Put $s=-2$	
		$\therefore 1=B(-2-3)$	
		$\therefore B=\frac{-1}{5}$	½
		Put $s=3$	
		$\therefore 1=C(3+2)^2$	

Q. No.	Sub Q.N.	Answers	Marking Scheme
6.	c)	$\therefore C=\frac{1}{25}$	½
		Put $s=0$	
		$\therefore 1=A(-6)+B(-3)+C(4)$	
		$\therefore 1=-6A-\frac{1}{5}(-3)+\frac{1}{25}(4)$	
		$\therefore 1-\frac{3}{5}-\frac{4}{25}=-6A$	
		$\therefore \frac{6}{25}=-6A$	
		$\therefore A=\frac{-1}{25}$	½
		$\therefore \frac{1}{(s+2)^2(s-3)}=\frac{\frac{-1}{25}}{s+2}+\frac{\frac{-1}{5}}{(s+2)^2}+\frac{\frac{1}{25}}{s-3}$	
		$L^{-1}\left\{\frac{1}{(s+2)^2(s-3)}\right\}=L^{-1}\left\{\frac{\frac{-1}{25}}{s+2}+\frac{\frac{-1}{5}}{(s+2)^2}+\frac{\frac{1}{25}}{s-3}\right\}$	
		$=\frac{-1}{25}e^{-2t}-\frac{1}{5}t\cdot e^{-2t}+\frac{1}{25}e^{3t}$	2
		--	
		Important Note	
		In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.	
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